Exam Game Theory & Auctions & Voting
(Web Sciences 201500025)

Friday January 20, 2017, 8.45-11.45

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of five problems. Please start a new page for every problem.
- You have 3 h time.
- Please write name and student ID on your solutions.
- Total number of points: 36+4 = 40. Distribution of points:

1a: 2	2a: 3	3a: 3	4a: 2	5a: 3
1b: 2	2b: 3	3b: 3	4b: 2	5b: 4
1c: 2	:	3c: 1	4c: 1	5c: 5

Exercise 1

Decide for each of the following statements whether it is true or false. Give a short argument to justify your answer (one or two sentences, or a counterexample).

(a) *(2 points)* For every two-player game, the following holds: If none of the two players has a dominating strategy, then there are no pure Nash equilibria.

(b) *(2 points)* Any network routing game (with finitely many players) has a pure Nash equilibrium.

(c) *(2 points)* A second-price sealed bid auction can be modelled as a bimatrix game.

Exercise 2

Consider the game specified by the following payoff matrix and answer the following questions.

		opponent		
		ℓ	m	r
	T	2/1	4/6	0/3
you	M	1/4	5/1	3/2
	B	3/1	5/1	1/3

(a) *(3 points)* What, if any, are your dominant, and strictly dominant strategies? What, if any, are your opponent's dominant, and strictly dominant strategies?

(b) *(3 points)* Which strategies are strictly dominated by which strategies? Write down the resulting game with the strictly dominated strategies removed.

Does the resulting reduced game have strictly dominated strategies? If yes, by which strategies are they dominated? Iterate the process of removing strictly dominated strategies until no strictly dominated strategy remains.

Exercise 3

Consider the example of the following matching market.

		items		
		A	B	C
buyers	W	11	7	5
	X	8	8	6
	Y	4	3	0

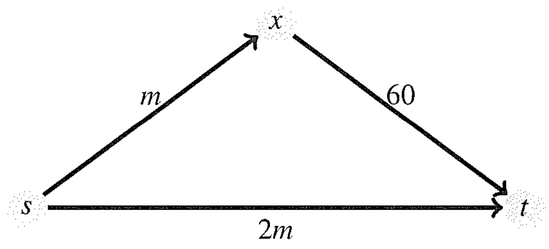
(a) *(3 points)* Run the price raising procedure to compute market-clearing prices.

(b) *(3 points)* Run the VCG procedure to compute VCG prices. (You do not have to compute all $V_{-?}^{-?}$ values, but only the three relevant ones.)

(c) *(1 point)* Compare the resulting assignment of items to buyers with the outcome in (a) and comment on that.

Exercise 4

Consider the following simple road network. There are 300 cars that want to go from s to t. The travel time on each road is indicated in the figure below.
(On the arc from s to x it takes m minutes if there are m cars on that road. Along the arc from s to t it takes $2m$ minutes in case there are m cars on that road. For the arc from x to t the travel time is 60 minutes independently of the number of cars using this road.)



(a) *(2 points)* Compute the social optimum for this road network.

(b) *(2 points)* Compute a Nash equilibrium for this road network.

(c) *(1 points)* What can you conclude for the price of anarchy for this network routing game?

Question 5

(a) *(3 points)* Consider a voting rule on $m \geq 3$ alternatives in which voters with complete and transitive preferences distribute the scores 5, 3 and 1 over their three most favoured alternatives.

Prove or give a counterexample: If the Condorcet winner exists, then this alternative wins.

(b) *(4 points)* Suppose that you receive the assignment to design a voting rule F that fulfills two requirements: it should be unanimous (meaning that if $X \succ_i Y$ for all voters i in a preference profile P, then $X \succ Y$ in $F(P)$) and it should be anonymous (meaning that $F(\succ_1, \ldots, \succ_n) = F(\succ_{\pi(1)}, \ldots, \succ_{\pi(n)})$ for any permutation π of the voters). We assume for simplicity that the number of voters is odd.

Give a voting rule that fulfills both requirements. Also give at least one major weakness of *any* voting rule that fulfills these two requirements.

(c) *(5 points)* Consider the following experiment. There are two urns, one with 10 red balls and one with 1 red and 9 black balls. Let us call the first urn red and the second one black. Any of the two urns is chosen with probability $\frac{1}{2}$. Once this has been done, there are 3 players who get to see, each individually, one random ball from the urn. In order to decide if the urn is actually red or black, the players must vote red or black *by majority*. Assume the players are non-strategic (i.e. they follow their private signals).

(i) What is the probability that a player sees a red ball?

(ii) What is the probability that the players decide red? (Hint: the result is $> \frac{1}{2}$.)

(iii) Suppose the players have decided red. What is the conditional probability that the urn is indeed red?

(iv) Briefly discuss: When a player sees a red ball and is interested in the correct outcome, should he actually be non-strategic and follow the signal?