Kenmerk : TW2014/DWMP/024/ha

Course : **Discrete Mathematics for Computer Science**
Date : October 21, 2014
Time : 10.45-12.30 hrs

Motivate all your answers.
The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. Let $\mathcal{U}$ be a universe and let $A, B \subseteq \mathcal{U}$ be sets.
Furthermore let I be a nonempty index set and let, for each $i \in I$, $A_i \subseteq \mathcal{U}$ be a set.
Give quantified expressions for the following statements.

(a) [3 pt] $\overline{A} \cap B = \varnothing$.

(b) [3 pt] $\bigcup_{i \in I} A_i = \mathcal{U}$.

2. [6 pt]
Prove the following equivalence using the "Laws of Logic".

$$((p \to q) \land (\neg q \land (r \lor \neg q))) \iff \neg(p \lor q).$$

3. [6 pt]
Let A, B and C be sets in a universe $\mathcal{U}$.
Prove that: $A - (B \cup C) = (A - B) \cap (A - C)$.

4. [6 pt]
Let the sequence of numbers $a_1, a_2, a_3, \ldots$ be given by:

$a_1 = 3$, $a_2 = 12$, and for $n \geq 3$: $a_n = 2a_{n-1} + 5a_{n-2}$.

Prove with mathematical induction that for all $n \in \mathbb{Z}^+$, $a_n \leq \left[\dfrac{7}{2}\right]^n$.

5. [6 pt]
Let A and B be sets, $f : A \to B$ a function and $A_1, A_2 \subseteq A$.
Prove that if f is one-to-one, then $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$.

6. [6 pt]
Let $A = \{1, 2, 3, 4, 5, 6\}$ and let R be the relation on A given by:

$$xRy \text{ if and only if } x^2 - y^2 \text{ is divisible by } 3.$$

Show that R is an equivalence relation on A and determine the partition of A induced by R.

Total: 36 points