Mathematics A, Solution/Correction standard, Sample Test 1

1. (a) $A_2 = [-\frac{1}{2}, 4)$, $A_4 = [-\frac{1}{4}, 8)$ **[0.5 pt]**

   So $A_4 - A_2 = [4, 8)$. **[0.5 pt]**

   $A_1 = [-1, 2)$, $A_2 = [-\frac{1}{2}, 4), \ldots, A_{10} = [-\frac{1}{10}, 20)$. **[0.5 pt]**

   So $\bigcap_{k=1}^{10} A_k = [\frac{1}{10}, 2)$ and $\bigcup_{k=1}^{10} A_k = [-1, 20)$. **[1.5 pt]**

   (Argumentation: **[1 pt]**, answers: **[2 pt]**).

   (b) Truth table for $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$:

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	0	0
0	1	1	0	1	1	1	1
1	0	0	0	1	0	1	0
1	0	1	0	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

[2 pt]

   Conclusion: The fifth and last column are not identical, so the propositions are not logial equivalent. **[1 pt]**

   (Correct truth table **[2 pt]** (column incorrect: -0.5 pt), explanation how the conclusion can be deduced from the table + correct answer **[1 pt]**. If table is not correct but the way the conclusion is deduced from the table is: 1 pt).

2. Basis step for $n = 1$:

   $\sum_{i=1}^{1} (-1)^i \cdot i^2 = (-1)^1 \cdot 1^2 = -1$ and also $\frac{1}{2}(-1)^1 \cdot 1(1+1) = -1$.

   So the statement is correct for $n = 1$. **[0.5 pt]**

   Induction step:

   Let $k \geq 1$ and suppose that:

   $\sum_{i=1}^{k} (-1)^i \cdot i^2 = \frac{1}{2}(-1)^k k(k+1)$ (Induction hypothesis: IH). **[1 pt]**

   We must show that IH implies: $\sum_{i=1}^{k+1} (-1)^i \cdot i^2 = \frac{1}{2}(-1)^{k+1}(k+1)(k+2)$. **[0.5 pt]**

Well: $\sum_{i=1}^{k+1}(-1)^i \cdot i^2 = \sum_{i=1}^{k}(-1)^i \cdot i^2 + (-1)^{k+1} \cdot (k+1)^2$. **[0.5 pt]**

By IH this expression is equal to $\frac{1}{2}(-1)^k k(k+1) + (-1)^{k+1} \cdot (k+1)^2$. **[0.5 pt]**

Now it remains to show that

$\frac{1}{2}(-1)^k k(k+1) + (-1)^{k+1} \cdot (k+1)^2 = \frac{1}{2}(-1)^{k+1}(k+1)(k+2)$. **[0.5 pt]**

Indeed, dividing this equation by $\frac{1}{2}(-1)^k(k+1)$, we obtain:

$k - 2(k+1) = -(k+2)$, which is obviously correct. **[0.5 pt]**

Now we obtain from the principle of mathematical induction that for all $n \geq 1$:

$$\sum_{i=1}^{n}(-1)^i \cdot i^2 = \frac{1}{2}(-1)^n n(n+1).$$

(From the proof it must be crystal clear whàt is supposed **[1 pt]** and whàt must be proved **[1 pt]**. In case of nonsense formulations like “Suppose it is correct FOR ALL n, so it also holds for $n+1$”: at most 1 pt for the entire exercise)

3. (a) From the $20 + 30 = 50$ members, a set of 7 members must be selected, and the order in which these members are selected does not matter. So we want to know the number of 7-combinations in 50. **[0.5 pt]**

   The number is equal to: $\binom{50}{7}$. **[0.5 pt]**

   (answer: **[0.5 pt]**, (some) argumentation: **[0.5 pt]**).

   (b) The number of women must be 4, 5, 6 of 7.

   The number of ways a group of 4 women can be chosen out of 30 is equal to:

   $$\binom{30}{4}.$$ **[0.5 pt]**

   For *each* choice of 4 women, there are $\binom{20}{3}$ ways the complete the committee with 3 men.

   So, the number of committees with exactly 4 women is:

   $$\binom{30}{4} \cdot \binom{20}{3}.$$ **[1 pt]**

   Similarly, the number of committees with 5, 6 en 7 women respectively is equal to:

   $$\binom{30}{5} \cdot \binom{20}{2}, \quad \binom{30}{6} \cdot \binom{20}{1} \quad \text{and} \quad \binom{30}{7} \cdot \binom{20}{0}.$$ **[0.5 pt]**

   Therefore, the total number of committees that can be formed is:

   $$\binom{30}{4} \cdot \binom{20}{3} + \binom{30}{5} \cdot \binom{20}{2} + \binom{30}{6} \cdot \binom{20}{1} + \binom{30}{7} \cdot \binom{20}{0}.$$ **[1 pt]**

   (Just the answer, without any argumentation: **[1.5 pt]**).