Pearl 000 of Computer Science (201300070)

9 September 2016, 13:45–14:45

- You are allowed to use one A4-sized sheet of paper with your own notes at this exam, and a *simple* calculator.
- Scientific or graphical calculators, laptops, cell phones, books etc. are not allowed. ***Put them in your bag right now!***
- The number of points per question is indicated in the margin.

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1. Binary numbers

(a) Convert the 2-complement binary number 100100 to decimal. Show how you calculate this. [6]

(b) Convert the decimal number 100 to hexadecimal. Explain. [6]

(c) Suppose you have an 8-bit 2-complement signed binary number, and want to multiply this by 2. This can be done by shifting the bits: in which direction, by how many positions, and what should be substituted in the free position(s)? Is any special treatment needed for the left-most bit, since this bit describes the sign of the number? Explain. [7]

(d) Suppose you convert an n-bit binary number to decimal, assuming it was an unsigned number. However, you later find out that that binary number was actually meant as a 2-complement signed number. How large is the error you made? Explain. [6]

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2. Boolean logic

(a) Give the truth table of a “parity calculation” with three inputs: the output is such that the total number of 1s among the inputs and the output is even. [6]

(b) Suppose you have two NOR gates (with two inputs each) and feed their outputs to a NAND gate. What kind of four-input gate results from this (AND, NOR, etc.)? Explain. [5]

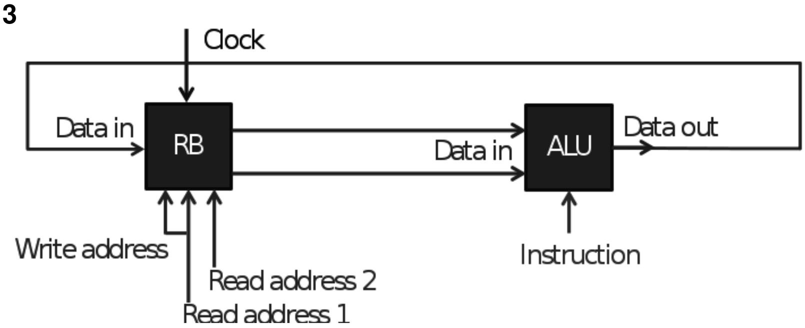
(c) Consider the following derivation in Boolean algebra. Indicate for each (numbered) equals sign which rule is applied; choose from: “commutative”, “identity”, “complement”, “distributive”, “DeMorgan”, or “wrong”; the latter if you think that that step is not correct. (It is possible that a rule is used multiple times, or not at all, in this derivation.) [8]

$$AB+A\overline{B}+\overline{B}\,\overline{A} \overset{(1)}{=} (A\cdot(B+\overline{B}))+\overline{B}\,\overline{A} \overset{(2)}{=} (A\cdot 1)+\overline{B}\,\overline{A} \overset{(3)}{=} A+\overline{B}\,\overline{A} \overset{(4)}{=} (A+\overline{B})\cdot(A+\overline{A}) \overset{(5)}{=} (A+\overline{B})\cdot 1 \overset{(6)}{=} A+\overline{B}$$

(d) Sketch how one can realise the following formula using only NAND gates: [6]
$A + (B \cdot \overline{C})$

Continued on next page...

3. Problem 3 20



The ALU of the processor above has two instructions: 0 = 'OR' and 1 = 'AND'. Furthermore it has 4 1-bit registers. The starting value for register R4 equals 0. Give for this processor the program for the following computation: R4 = R1 AND (R2 OR R3)

	read address 1 / write address	read address 2	instruction
Timeslot 0			
Timeslot 1			
Timeslot 2			
Timeslot 3			
Timeslot 4			
...			

4. Problem 4

```
LDI   R17, $03
LDI   R18, $02
LDI   R19, $01
LDI   R20, $00
MUL   R18, R17
ADD   R20, R0
DEC   R20
DEC   R17
MOV   R21, R17
SUB   R21, R19
BRNE  -7
```

Given this AVR program; "BRNE" means "BRanch if Not Equal", "DEC" means "Decrement (subtract 1)", "SUB" means "Subtract", "MUL" means unsigned MULtiply, where the least significant part of the result will be placed in R0.

Assume that each instruction takes 1 clock cycle, except jumping to a different address, which takes 2 clock cycles.

(a) What value (in decimal representation) does register R20 contain after executing this program? Show how you come to the answer. 10

(b) How many clockcycles does this program take? Explain. 10

5. Problem 5 10

What is the mathematical function that is computed by the code below?
Write as a function of X and Y, e.g. f(X, Y) = X + Y.
Assumption: X and Y are larger than 0 and the result is available in R20.

```
LDI   R17, $X
LDI   R18, $Y
LDI   R19, $00
LDI   R20, $00
MOV   R19, R18
ADD   R20, R17
DEC   R18
BRNE  -3
ADD   R20, R19
DEC   R17
BRNE  -3
```