Solutions Mathematics B2 Sample Test 1 (Newton)

Kenmerk: TW2013/MathB2/SampleTest1

1.

$$\lim_{x\to\infty}\left(\frac{2x^2+4x-1}{x^2+7x}\right)^{10} = \left(\lim_{x\to\infty}\frac{2x^2+4x-1}{x^2+7x}\right)^{10} = \left(\lim_{x\to\infty}\frac{2+\frac{4}{x}-\frac{1}{x^2}}{1+\frac{7}{x}}\right)^{10} = 2^{10}$$

2.

$$f(x) = \begin{cases} x^2+2x+1 & \text{for } x \le 0 \\ 1-\sqrt{x} & \text{for } x > 0 \end{cases}$$

(a)

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x^2+2x+1) = 1$$
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(1-\sqrt{x}) = 1$$

So $\lim_{x\to 0} f(x)$ exists, equals 1, and $f(0) = 1$; so $f(x)$ is continuous at $x = 0$.

(b) For $x < 0$ we have $f'(x) = 0 \Leftrightarrow 2x + 2 = 0 \Leftrightarrow x = -1$.
For $x > 0$ we have $f'(x) = 0 \Leftrightarrow -\dfrac{1}{2\sqrt{x}} = 0$, so here are no solutions.
Critical points are $x = -1$ and $x = 0$
endpoints $x = -3$ and $x = 2$
$f(-3) = 4$, $f(-1) = 0$,$f(0) = 1$ and $f(2) = 1 - \sqrt{2}$
We conclude: The absolute maximum is 4 and the absolute minimum is $1 - \sqrt{2}$

3.

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x}$$
$$\lim_{x\to 0^+} x\ln x = \text{"}0\cdot -\infty\text{" undetermined}$$

Make that we can apply l'Hôpital's rule:

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+}\frac{\ln(x)}{\frac{1}{x}} = \left(\text{"}\frac{-\infty}{\infty}\text{", use l'Hôpital}\right)$$
$$\lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} -x = 0, \text{ therefore } \lim_{x\to 0^+} x^x = e^0 = 1$$

4.

$$f(x) = \begin{cases} \frac{xy}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$$

(a) f is continuous at (x_0, y_0) by definition:

i. f is defined at (x_0, y_0)

ii. $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ exists

iii. $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0, y_0)$

f is not continuous at $(0,0)$ for since if we approach $(0,0)$ first by the curve $x = y$ (diagonal) and secondly by the line $y = 0$ (x-axis) we get different limits:

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} \frac{x^2}{x^2+x^2} = \frac{1}{2}$$

respectively

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} \frac{0}{x^2+0} = 0$$

So $\lim\limits_{(x,y)\to(0.0)} f(x,y)$ does not exist.

(b)

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$
$$\frac{\partial f}{\partial y}(0,0) = \lim_{h\to 0} \frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

(c) Tangent plane is:

$$f_x(1,2)(x-1) + f_y(1,2)(y-2) - (z - \frac{2}{5}) = 0$$
$$f_x = \frac{\partial f}{\partial x} = \frac{y(x^2+y^2) - 2x \cdot xy}{(x^2+y^2)^2}, \text{ so } f_x(1,2) = \frac{6}{25}$$
$$f_x = \frac{\partial f}{\partial y} = \frac{x(x^2+y^2) - 2y \cdot xy}{(x^2+y^2)^2}, \text{ so } f_y(1,2) = \frac{-3}{25}$$
$$\text{answer: } \frac{6}{25}(x-1) + \frac{-3}{25}(y-2) - (z - \frac{2}{5}) = 0$$