Motivate all your answers.
The use of electronic devices is not allowed.

1. [4 pt]

   For $k \in \{2, \ldots, 10\}$, the set A_k is given by: $\quad A_k = \left\{ \dfrac{1}{k}, \dfrac{2}{k}, \ldots, \dfrac{k}{k} \right\}.$

   Determine $\quad A_4 \cap A_6; \quad A_4 \cup A_6; \quad A_4 - A_6 \quad$ and $\quad \displaystyle\bigcap_{k=2}^{10} A_k.$

2. [2 pt]
   Let A and B be sets.
   A *quantified statement* for $A \cap B = \varnothing$ is: $\quad \neg \exists x \, (x \in A \land x \in B).$

   Give a quantified statement for $\quad \overline{A} \subseteq B.$

3. (a) [2 pt] Prove that for all $x, y \in \mathbb{R}$ the following inequality holds:

   $$||x| - |y|| \leq |x| + |y|.$$

   Hint: give a proof by cases.

   (b) [3 pt] Prove with mathematical induction that for all $n \geq 1$,

   $$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

4. Consider a deck of 52 cards: 13 hearts, 13 spades, 13 clubs and 13 diamonds.

   (a) [1 pt] There are 13 children and each child is given one card. In how many ways can this be done?

   (b) [2 pt] In how many ways can one select a hand of 13 cards containing exactly 5 hearts and 5 spades?

Total: 14 points