Mathematics C (Cayley) Code : AM 734517
Date : July 6, 2018
Time : 13.45 – 15.45 hrs

First read these instructions carefully:

This test contains 9 exercises. The complete solutions of Exercises 5, 6b and 7 must be accurately written down on a separate sheet, including calculations and argumentation.
For the other exercises you are only required to fill in the final answers on the answer sheet at the end of this test. You must hand in this answer sheet as well as your hand written solutions to Exercises 5, 6b and 7.
The use of electronic devices is not allowed.

1. *Fill in your final answer to this exercise on the separate answer sheet!*

   Given is the following matrix equation with unknown $X \in \mathbb{R}^{2\times 2}$:

   $$A^{\mathrm{T}}X + XA = B,$$

   where

   $$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}.$$

   The unknown matrix X has the following form:

   $$X = \begin{pmatrix} x_1 & x_2 \\ x_2 & x_3 \end{pmatrix}$$

   This leads to a set of linear equations in x_1, x_2 and x_3.

   Determine all possible x_1, x_2 and x_3 for which the above matrix equation is satisfied.

2. *Fill in your final answers to this exercise on the separate answer sheet!*

   The matrix A is given by:

   $$A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & -3 & 2 \\ 3 & -4 & 1 \end{pmatrix}$$

   a) Determine a basis for Null A.
   b) Determine a basis for Col A.

3. *Fill in your final answer to this exercise on the separate answer sheet!*

   Given are four vectors in $\mathbb{R}^3$:

   $$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \qquad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \qquad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{v}_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

   In this case,

   $$\mathcal{T} = \text{Span}\,\{\, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \,\}$$

   is a basis for $\mathbb{R}^3$. Determine $[\mathbf{v}_4]_{\mathcal{T}}$.

4. *Fill in your final answer to this exercise on the separate answer sheet!*

   Given is the matrix

   $$A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}.$$

   Determine, if it exists, the inverse of the matrix A.

5. The distance between two vectors in $\mathbb{R}^3$ is given by:

$$\|\mathbf{x} - \mathbf{y}\| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$$

with

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

Given are two vectors:

$$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} \alpha \\ 2 \\ -1 \end{pmatrix}$$

We look at the set $\mathcal{X}$ of all vectors for which the distance to $\mathbf{v}$ is equal to the distance to $\mathbf{w}$. In other words:

$$\mathcal{X} = \left\{ \mathbf{x} \in \mathbb{R}^3 \mid \|\mathbf{x} - \mathbf{v}\| = \|\mathbf{x} - \mathbf{w}\| \right\}$$

   a) Show that $\mathcal{X}$ is a linear subspace of $\mathbb{R}^3$ when $\alpha = 1$ or $\alpha = -1$.
   b) Show that $\mathcal{X}$ is **not** a linear subspace of $\mathbb{R}^3$ when $\alpha \neq 1$ and $\alpha \neq -1$.

6. $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation which takes each point $(x_1, x_2) \in \mathbb{R}^2$ and rotates it first through 45 degrees (counterclockwise) and then projects the result on the line $y = x$.

   a) *Fill in your final answer to this exercise on the separate answer sheet at the end of this test!*

      Determine the representation matrix of T.
   b) Determine whether T is surjective (onto) and/or injective (one-to-one).

7. You are given that the matrix A is invertible and diagonalizable. Show that A^{-1} is also diagonalizable.

8. *Fill in your final answer to this exercise on the separate answer sheet at the end of this test!*

   Determine the volume of the parallelepiped with vertices $(0, 0, 0)$, $(0, -2, 2)$, $(-5, 0, -4)$ and $(3, 0, 4)$.

9. *Fill in your final answer to this exercise on the separate answer sheet at the end of this test!*

   Consider the matrix

$$A = \begin{pmatrix} 1 - 2\alpha & -2\alpha & \alpha \\ \alpha + 1 & \alpha + 2 & -1 \\ 2 - 2\alpha & 2 - 2\alpha & 2\alpha - 1 \end{pmatrix}$$

   with $\alpha \in \mathbb{R}$. Determine all $\alpha \in \mathbb{R}$ for which A has eigenvalue 1 with a corresponding eigenspace with dimension 1.

For the exercises the following number of points can be obtained:

Exercise	1.	3 points	Exercise	2.	3+3 points	Exercise	3.	3 points
Exercise	4.	3 points	Exercise	5.	3+3 points	Exercise	6.	3+2 points
Exercise	7.	3 points	Exercise	8.	3 points	Exercise	9.	3 points

The grade is determined by adding 4 points to the total number of points obtained and dividing by 4.

Answer sheet
Antwoordenblad

Legibly fill in your answers in the corresponding box
Schrijf je antwoord leesbaar in de bijbehorende box

Math C (AM 734517) July 6, 2018

Name/Naam: ....

Programme/Opleiding: ....

1.

2. a)

b)

3.

4.

6. a)

8.

9.