Questions

1	2	3	4	5

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Intro to Math 19/20
Introduction to Mathematics - Sample Test 1
6 September 2019 09:00 - 12:00

There are three types of questions: "final answer", "open answer" and "multiple choice".

- **Final answer**

  In the text frame below the question, you only provide *one* answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

- **Open answer**

  You provide a calculation or motivation in the text frame below the question. The calculation or motivation will be graded. Any text outside the frame will be ignored.

- **Multiple choice**

  Only one answer is correct. Choose the correct answer by marking it with a black or blue pen or pencil.

- **Multiple response**

  More than one answer may be correct. Choose the correct answer(s) by marking them with a black or blue pen or pencil.

Before you enter a solution into a text frame, elaborate the answer on scratch paper. Do not submit scrap paper.

Exercise 1: Sets

Let $A = \{\frac{k}{k+1} \mid k \in \mathbb{N}\}$. For each of the following, either write 'does not exist or give its value.

Provide the answer (and only the answer) in the frames below.

0.5p **1a** $\inf A =$

0.5p **1b** $\max A =$

0.5p **1c** $\min A =$

0.5p **1d** $\sup A =$

Exercise 2: Predicate logic

For each of the following predicates, determine whether it is a tautology, a contradiction or neither.

These are multiple choice questions; per question, only one answer is correct.

Grading:
4 correct answers: 2 pt
3 correct answers: 1 pt
2 or less correct answers: 0 pt

The amount of points stated in the margin is the maximum *amount of points that can be scored for that question.*

0.5p **2a** $p \wedge p$

- ○ tautology
- ○ contradiction
- ○ neither

0.5p **2b** $p \wedge \neg p$

- ○ tautology
- ○ contradiction
- ○ neither

0.5p **2c** $p \vee \neg p$

- ○ tautology
- ○ contradiction
- ○ neither

0.5p **2d** $p \rightarrow \neg p$

- ○ tautology
- ○ contradiction
- ○ neither

Exercise 3: Even and odd

3p **3** Let $m \in \mathbb{Z}$ and $n \in \mathbb{Z}$. Prove that if the product mn is even, then either m is even, or n is even (or both).

Give a full calculation/motivation for your answer in the frame below.

Exercise 4: Mathematical induction

3p **4** Use mathematical induction on n to prove that $\forall n \in \mathbb{N} \cup \{0\}$:

$$\sum_{i=0}^{n} \binom{i+2}{i} = \binom{n+3}{n}.$$

Hint: use that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ for all $n \in \mathbb{N}$, $r \in \mathbb{N}$, $r \leqslant n$.

Give a full calculation/motivation for your answer in the frame below.

Exercise 5: Combinatorics

Consider the set A of numbers consisting of 4 digits, where each digit is from the set $\{1, 2, 3\}$. For example: $1311 \in A$.

For each question, give a full calculation/motivation for your answer in the frame below the question.

2p **5a** How many odd numbers are there in A?

2p **5b** How many numbers in A are either odd, or start with the digit 1, or both?