Discrete Mathematics for Computer Science Exam, Part 1 October 27, 2021

Provide explanation for all answers. Failing to do this would result in no points given.
Use of electronic devices is not allowed.
Answers to different problems must go in separate sheets
Time: one hour

1. (4 pts.) Write the statements below in formal logic:

   (a) Recall that $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$ is the set of nonnegative integers. Consider an open statement $p(n)$, where the free variable n is a nonnegative integer. The statement is:

   If $p(0)$ is true, and for any nonnegative integer k, $p(k)$ is true implies $p(k+1)$ is also true, then $p(n)$ is true for all nonnegative integers n.

   (b) Let L be a set of subsets from a universe $\mathcal{U}$, i.e., $L \subseteq \mathcal{P}(\mathcal{U})$. The statement is

   Each pair of elements from L is either disjoint or one is contained in the other.

2. (3 pts.) Using the rules of logic and inference, establish the validity or invalidity of the following argument:

$$\begin{array}{r} p \to q \\ \neg p \to r \\ r \to s \\ \neg q \\ \hline \therefore s \end{array}$$

3. (3 pts.) Let $A = \{a, b\}$ and $B = \{\{a, b\}, c\}$. Determine:

   (a) $\mathcal{P}(A)$.

   (b) $A \cap B$.

   (c) $A \times B$.