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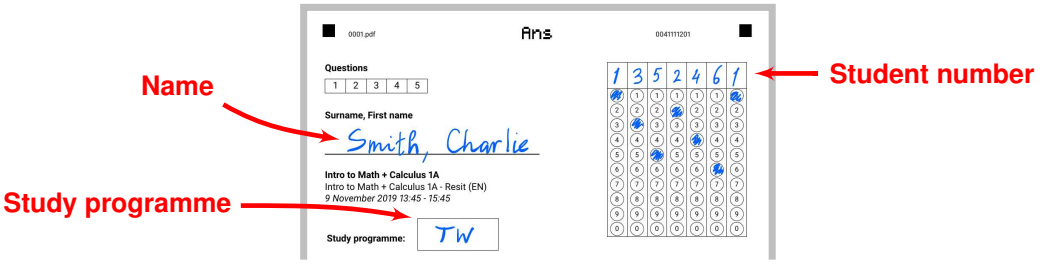
UNIVERSITY OF TWENTE.

Remove this page

Instructions

- This is a cover page. **Remove it before you submit your work**.
  You can use the flip side as scrap paper.
- Before you enter a solution, elaborate the answer on scrap paper. **Do not submit scrap paper**.
- Use a **blue** or **black** pen to fill in your answer. **Do not use a pencil**.
- Check your answer whenever possible.
- Enter your name (surname first), student number and study programme as follows:



Question types

Final answer

In the text frame below the question, you provide only *one* answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading. If you fill in an equation, only the right-most member will be regarded as the answer.

Open answer

You provide a calculation or motivation in the text frame below the question. The calculation or motivation will be graded. Any text outside the frame will be ignored.

Multiple choice

Uses round markers. Only one answer is correct. Choose the correct answer by marking it with a black or blue pen.

Multiple response

Uses square markers. More than one answer may be correct. Choose the correct answer(s) by marking them with a black or blue pen.

Correcting your answer

If you need to correct a multiple choice/response question, crossout the wrong answer, and mark the right answer with an arrow, like this:

Extra writing space

If you need more space, you can write in the frame provided at the end of the exam. Clearly refer to this space in the original exercise.

Questions

1	2	3	4	5	6	7	8	9	10
11									

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Surname, First name

Intro to Math + Calculus 1A
Intro to Math + Calculus 1A - Resit (EN) 23A
8 November 2019 13:45 - 16:45

Study programme:

- The use of a calculator or any other electronic device is not allowed.
- Please turn off your cell phone.

Exercise 1: Sets

2p **1** For each of the following statements, determine whether it is true or false for all sets A and B.

This is a multiple response question: more than one statement can be true. Check the boxes of the statements that you think are true, and leave the boxes of the false statements unchecked.

Grading:
4 correct answers: 2 pt
3 correct answers: 1 pt
2 or less correct answers: 0 pt

- ☐ If $A \subset B$, then for every element $a \in A$ it is true that $a \in B$.
- ☐ If for every element $a \in A$ it is true that $a \in B$, then $A \subset B$.
- ☐ If $A \subset B$, then A is an element of B.
- ☐ If $A \subset B$, then A is not an element of B.

Exercise 2: Proposition logic

2p **2** Calculate the truth table for the statement $(p \wedge \neg p) \leftrightarrow (p \rightarrow q)$.

Provide the answers (and only the answers) in the table below.

Grading:
4 correct answers: 2 pt
3 correct answers: 1 pt
2 or less correct answers: 0 pt

p	q	$(p \wedge \neg p) \leftrightarrow (p \rightarrow q)$
0	0	
0	1	
1	0	
1	1	

Exercise 3: Numbers

3p **3** Let $x \in \mathbb{Z}$. Prove that $x^2 + 5x + 1$ is odd.

Give a full calculation/motivation in the frame below.

Exercise 4: Mathematical induction

4p **4** Use mathematical induction on n to prove that $\forall n \in \mathbb{N}$:

$$\sum_{i=1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Give a full calculation/motivation in the frame below.

Exercise 5: Combinatorics

For the questions below, give full calculations/motivations/descriptions in the frames below the questions.

1.5p **5a** Use Newton's binomial theorem to find the coefficient of x^3y^6 in $(x+y)^9$ and evaluate this coefficient.

Simplify the answer as much as possible. Write the final answer down as one single number.

1.5p **5b** Use Newton's binomial theorem to find the coefficient of x^3y^6 in $(2x+3y)^9$.

You don't need to evaluate it. (For example, if the coefficient were $3!+4!$, then $3!+4!$ is a correct answer.)

Exercise 6: Vectors

Define the points $P(1,-1,-1)$, $Q(1,3,3)$ and $R(-1,-1,1)$. Let $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$.

1p **6a** Calculate $\mathbf{u} \times \mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

2p **6b** Calculate the angle between $\mathbf{u}$ and $\mathbf{v}$.

Provide the answer (and only the answer) in the frame below. The answer must be numerical: only numbers and constants, like e, π, and $\sqrt{2}$ are allowed. Express the angle in radians or degrees.

2p **6c** Calculate the projection of $\mathbf{u}$ onto $\mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

2p **6d** Calculate an equation of the plane through P, Q and R.

Provide the answer (and only the answer) in the frame below. Simplify the equation as much as possible.

Exercise 7: Limits and differentiability

2p **7a** Show that $1 - \sqrt{1-x^2} = \dfrac{x^2}{1+\sqrt{1-x^2}}$ for all $-1 < x < 1$.

Give a full calculation/motivation in the frame below.

2p **7b** Use **2a** to calculate $\lim\limits_{x \to 0^+} \dfrac{\sqrt{1-\sqrt{1-x^2}}}{x}$ and $\lim\limits_{x \to 0^-} \dfrac{\sqrt{1-\sqrt{1-x^2}}}{x}$, and use the results to show that $\lim\limits_{x \to 0} \dfrac{\sqrt{1-\sqrt{1-x^2}}}{x}$ does not exist.

Give a full calculation/motivation in the frame below and on the next page.

2p **7c** Prove that the function $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$ is not differentiable at 0.

Give a full calculation/motivation in the frame below.

Exercise 8: Extreme values

3p **8** Calculate the absolute minimum and maximum value of the function $f(x) = (3 - x^2)e^x$ on the interval $[-1, \sqrt{3}]$.

Give a full calculation/motivation in the frame below.

Exercise 9: Limits of Functions of Two Variables

Define $f(x,y) = \dfrac{x^3+y^3}{x^2+y^2}$ for all $(x,y) \neq (0,0)$.

3p **9** Calculate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ if this limit exists, or prove that this limit does not exist.

Give a full calculation/motivation for your answer in the frame below.

Exercise 10: Tangent Planes

3p **10** Define the function $f(x, y) = \ln(x + 2y - 2)$, and define $a = 1$ and $b = 1$. Let $c = f(a, b)$. Calculate an equation for the tangent plane V to the graph of f at the point (a, b, c).

Give a full calculation/motivation for your answer in the frame below.

Extra writing space

Clearly refer to this page if you use it.