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UNIVERSITY OF TWENTE.

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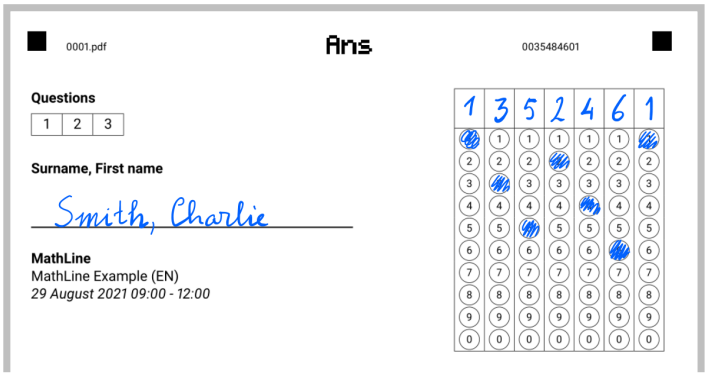
Instructions

- This is a cover page. **Remove it before you submit your work**.
  You can use the flip side as scrap paper.
- Before you enter a solution, elaborate the answer on scrap paper. **Do not submit scrap paper**.
- Use a blue or black pen to fill in your answer. **Do not use a pencil**.
- Check your answer whenever possible.
- Enter your name and student number as follows:



Question types

Final answer
In the text frame below the question, you provide only *one* answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer
You provide a calculation or motivation in the text frame below the question. The calculation or motivation will be graded. Any text outside the frame will be ignored.

Extra writing space

If you need more space, you can write in the frame provided at the end of the exam. Clearly refer to this space in the original exercise.

Questions

1	2	3	4	5

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Calculus 1A (Ca1A)
Calculus 1A - Test 1 (EN)
25 October 2019 13:45 - 15:45

Exercise 1: Vectors

Define the points $P(-1, 1, 2)$, $Q(1, 3, 1)$ and $R(2, 1, -1)$. Let $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$.

1p **1a** Calculate $\mathbf{u} \times \mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

2p **1b** Calculate the angle between $\mathbf{u}$ and $\mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

2p **1c** Calculate the projection of $\mathbf{u}$ onto $\mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

2p **1d** Calculate an equation of the plane through P, Q and R.

Provide the answer (and only the answer) in the frame below.

Exercise 2: Limits, differentiability and extreme values

1p **2a** Show that $\sqrt{1-\cos x} = \dfrac{\sqrt{\sin^2 x}}{\sqrt{1+\cos x}}$ for all $-\pi < x < \pi$.

Give a full calculation/motivation in the frame below.

2p **2b** Use **2a** to calculate $\lim\limits_{x\to 0^+} \dfrac{\sqrt{1-\cos x}}{x}$ and $\lim\limits_{x\to 0^-} \dfrac{\sqrt{1-\cos x}}{x}$, and use the results to show that $\lim\limits_{x\to 0} \dfrac{\sqrt{1-\cos x}}{x}$ does not exist.

Give a full calculation/motivation in the frame below.

2p **2c** Prove that the function $f(x) = \sqrt{1 - \cos x}$ is not differentiable at 0.

Give a full calculation/motivation in the frame below.

2p **2d** For which x in the interval $[-\frac{1}{2}\pi, \frac{3}{2}\pi]$ is $f'(x) = 0$?

Provide the solution(s) (and only the solution(s)) in the frame below.

Determine the absolute extreme values of $f(x) = \sqrt{1 - \cos x}$ on the interval $[-\frac{1}{2}\pi, \frac{3}{2}\pi]$.

1p **2e** *Fill the minimum value of $f(x)$ in in the frame below (only provide the answer).*

1p **2f** *Fill the* maximum *value of $f(x)$ in in the frame below (only provide the answer).*

Exercise 3: Limits of Functions of Two Variables

Define $f(x,y) = \dfrac{xy}{\sqrt{x^4+y^4}}$ for all $(x,y) \neq (0,0)$.

3p **3** Calculate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ if this limit exists, or prove that this limit does not exist.

Give a full calculation/motivation for your answer in the frame below.

Exercise 4: Tangent Planes

3p **4** Define the function $f(x, y) = x^3 - xy - y^2$, and define $a = 1$ and $b = -1$. Let $c = f(a, b)$. Calculate an equation for the tangent plane V to the graph of f at the point (a, b, c).

Give a full calculation/motivation for your answer in the frame below.

Extra writing space

Clearly refer to this page if you use it.