Questions

1	2	3	4	5

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Calculus 1A (Ca1A)
Calculus 1A - Sample Test 1 (EN)
9 November 2019 09:00 - 12:00

There are four types of questions: "final answer", "open answer", "multiple choice" and "multiple response".

- **Final answer**

  In the text frame below the question, you only provide *one* answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

- **Open answer**

  You provide a calculation or motivation in the text frame below the question. The calculation or motivation will be graded. Any text outside the frame will be ignored.

- **Multiple choice**

  Only one answer is correct. Choose and mark the correct answer.

- **Multiple response**

  More than one answer may be correct. Choose and mark the correct answer(s).

Exercise 1: Vectors

Define the vectors $\mathbf{u} = \langle -1, 2, 2 \rangle$ and $\mathbf{v} = \langle 4, -3, 0 \rangle$.

1p **1a** Calculate $\mathbf{u} \times \mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

2p **1b** Calculate $\cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

2p **1c** Calculate the projection of $\mathbf{u}$ onto $\mathbf{v}$.

Provide the answer (and only the answer) in the frame below.

Exercise 2: Equation of a Plane

Define the points $P(3, 2, -1)$, $Q(1, -1, 2)$ and $R(2, 2, 2)$.

3p **2** Calculate an equation of the plane through P, Q and R.

Provide the answer (and only the answer) in the frame below.

Exercise 3: Limits and continuity

3p **3a** Show that $\lim_{x \to 0} \dfrac{x}{\sqrt{x+1}-1} = 2$.

Give a full calculation/motivation in the frame below.

2p **3b** For which value of p is the following function continuous for every $x \in \mathbb{R}$?

$$f(x) = \begin{cases} 2x - p & \text{if } x \leqslant 0, \\ \dfrac{x}{\sqrt{x+1}-1} & \text{if } x > 0. \end{cases}$$

Provide the answer (and only the answer) in the frame below.

Exercise 4: Extreme Values

Define the function f as follows:

$f(x) = x^3 - x^2 - x + 1.$

2p **4a** Calculate the critical points of f.

Give a full calculation/motivation for your answer in the frame below.

1p **4b** Determine the absolute extreme values of $f(x)$ where $x \in [-2, 2]$.

This a multiple choice question; only one answer is correct.

- ○ Absolute minimum is -9, absolute maximum is 3.
- ○ Absolute minimum is -2, absolute maximum is 2.
- ○ Absolute minimum is -9, absolute maximum is $\frac{32}{27}$.
- ○ Absolute minimum is 0, absolute maximum is $\frac{32}{27}$.
- ○ Absolute minimum is $\frac{32}{27}$, absolute maximum is 3.
- ○ Absolute minimum is $-\frac{1}{3}$, absolute maximum doesn't exist.
- ○ Absolute minimum is $-\frac{1}{3}$, absolute maximum is $\frac{32}{27}$.
- ○ Absolute minimum is $-\frac{32}{27}$, absolute maximum is 3.

Exercise 5: Functions of Two Variables

Define $f(x,y) = \dfrac{x-y}{x^2+y^2}$ for all $(x,y) \neq (0,0)$.

3p **5a** Calculate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ if this limit exists, or prove that this limit does not exist.

Give a full calculation/motivation for your answer in the frame below.

3p **5b** Define $a = 1$ and $b = -1$. Let $c = f(a, b)$. Calculate an equation for the tangent plane V to the graph of f at the point (a, b, c).

Give a full calculation/motivation for your answer in the frame below.

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