Calculus 1B Test

10 January 2020 13:45 – 15:45

Question 1 (2p)

The expression $\sum_{k=1}^{n} \frac{2}{n\left(1+\frac{2k}{n}\right)^2}$ is a Riemann sum for a function in an interval.

Decide which integral equals the limit of this expression if n tends to ∞.

A $\int_0^1 \frac{2}{(1-2x)^2} dx$

B $\int_0^2 \frac{2}{(1+2x)^2} dx$

C $\int_0^2 \frac{2}{(1+x)^2} dx$

D $\int_0^1 \frac{2}{(1-x)^2} dx$

E $\int_0^2 \frac{2}{(1-x)^2} dx$

F $\int_0^2 \frac{2}{(1+2x)^2} dx$

G $\int_0^1 \frac{2}{(1+2x)^2} dx$

H $\int_0^1 \frac{2}{(1+2x)^2} dx$

Question 2 (3p)

Determine $\frac{dy}{dx}$ in case $y = \int_x^{x^3} e^{t^2} \sin(t)\, dt$.

Question 3 (2p)

Determine $\int \cos(x) \sqrt[5]{2 + \sin(x)} dx$.

Question 4 (5p)

Compute $\int_0^\infty \frac{x}{e^{2x}}$.

Question 5 (3p)

Compute $\sum_{n=0}^{\infty} \frac{2^n + 3^n}{6^n}$.

Question 6 (3p)

Find the Taylor polynomial of order 4 generated by $f(x) = \frac{1}{x}$ at $a = 2$.

Question 7 (3p)

Solve the initial value problem.

$$\begin{cases} \frac{dy}{dx} + \frac{3}{x} y = \frac{\sin(x)}{x^2} \\ y\left(\frac{\pi}{2}\right) = 1 \end{cases}$$

Question 8 (2p)

A population $P(t)$ is modeled by the initial value problem below. For which values of c is the population increasing?

$$\begin{cases} \frac{dP}{dt} = 1.2P(1 - \frac{P}{4200}) \\ P(0) = c \end{cases}$$

Question 9 (3p)

Given $z_1 = 3e^{\frac{\pi i}{4}}$ and $z_2 = 4e^{\frac{-\pi i}{6}}$. Find the Cartesian form for $z_1 z_2$.

A $12\sqrt{2} + 12i\sqrt{3}$

B $12\sqrt{2} - 12i\sqrt{3}$

C $\left(2\sqrt{6} + 2\sqrt{2}\right) + i(2\sqrt{6} + 2\sqrt{2})$

D $\left(2\sqrt{6} + 2\sqrt{2}\right) - i(2\sqrt{6} + 2\sqrt{2})$

E $\left(3\sqrt{6} - 3\sqrt{2}\right) + i(3\sqrt{6} + 3\sqrt{2})$

F $\left(3\sqrt{6} + 3\sqrt{2}\right) + i(3\sqrt{6} - 3\sqrt{2})$

G $12\sqrt{3} + 12i\sqrt{2}$

H $12\sqrt{3} - 12i\sqrt{2}$

Question 10 (3p)

Find all solutions in $\mathbb{C}$ of the equation $z^3 = i$.