Question 1

2p The expression $\sum_{k=1}^{n} \frac{k^2 + kn}{n^3}$ is a Riemann sum for a function on an interval. Decide which integral generates this Riemann sum

- ○ A) $\int_0^1 \frac{x^2+x}{x^3} dx$
- ○ B) $\int_0^n \frac{x^2+x}{x^3} dx$
- ○ C) $\int_0^1 \frac{x^2+x}{x^2} dx$
- ○ D) $\int_0^n \frac{x^2+x}{x^2} dx$
- ○ E) $\int_0^1 \frac{x^2+1}{x} dx$
- ○ F) $\int_0^1 (x^2 + 1) dx$
- ● G) $\int_0^1 (x^2 + x) dx$
- ○ H) $\int_0^n (x^2 + x) dx$

Question 2

Only write your final answer to the question in the box below.

2p Compute $\int_{-3}^{0} (1 + \sqrt{9 - x^2}) dx =$ $\boxed{3 + \frac{9}{4}\pi}$

Question 3

3p Determine $\frac{dy}{dx}$ in case $y = \int_0^{16x^4} (e^t \sqrt{t}) dt$

Choose from the alternatives below

- ○ A) $4x^2 e^{4x^2}$
- ○ B) $4x^4 e^{4x^4}$
- ● C) $256x^5 e^{16x^4}$
- ○ D) $256x^7 e^{16x^4}$
- ○ E) $64x^3 e^{4x^4}$
- ○ F) $64x^3 e^{16x^4}$
- ○ G) $256x^7 e^{4x^2}$
- ○ H) $256x^5 e^{4x^2}$

Question 4

Only write your final answer to the question in the box below.

2p Determine $\int (x\sqrt{1 + x^2}) dx =$ $\boxed{\frac{1}{3}(1+x^2)^{\frac{3}{2}} + C}$

or equivalent $\frac{1}{3}(1+x^2)\sqrt{1+x^2} + C$

($-\frac{1}{2}$ point if integration constant C is missing)

Question 5

4p Determine $\int_1^e (ln(x))^2 dx =$

We determine $\int (\ln(x))^2 dx$ applying partial integration

$\int (\ln(x))^2 dx = \int \ln^2(x) dx = x \ln^2(x) - \int x \, d \ln^2(x) =$

$= x \ln^2(x) - \int x \cdot 2 \ln(x) \cdot \frac{1}{x} dx = x \ln^2(x) - 2 \int \ln(x) dx$.

Using $\int \ln(x) dx = x \ln(x) - \int x \, d \ln(x) = x \ln(x) - \int x \cdot \frac{1}{x} dx =$

$= x \ln(x) - \int 1 dx = x \ln(x) - x$ we obtain

$\int (\ln(x))^2 dx = x \ln^2(x) - 2x \ln(x) + 2x$. Therefore

we have $\int_1^e (\ln(x))^2 dx = \left[x \ln^2(x) - 2x \ln(x) + 2x \right]_{x=1}^{e} =$

$= (e \ln^2(e) - 2e \ln(e) + 2e) - (1 \ln^2(1) - 2 \cdot 1 \ln(1) + 2 \cdot 1) =$

$= (e - 2e + 2e) - (0 - 0 + 2) = e - 2$

Question 6

5p Find the Taylor polynomial of order 3 generated by $f(x) = \sqrt{(1-x)}$ at $a = 0$

Firstly we compute the derivatives of f(x) at 0

$f(x) = \sqrt{1-x} = (1-x)^{1/2}$ $f(0) = 1$

$f'(x) = \frac{1}{2}(1-x)^{-\frac{1}{2}} \cdot -1 = \frac{-1/2}{\sqrt{1-x}}$ $f'(0) = -\frac{1}{2}$

$f''(x) = -\frac{1}{2} \cdot -\frac{1}{2}(1-x)^{-\frac{3}{2}} \cdot -1 = \frac{-1/4}{(1-x)^{3/2}}$ $f''(0) = -\frac{1}{4}$

$f'''(x) = -\frac{1}{4} \cdot -\frac{3}{2}(1-x)^{-\frac{5}{2}} \cdot -1 = \frac{-3/8}{(1-x)^{5/2}}$ $f'''(0) = -\frac{3}{8}$

Secondly we determine $P_3(x)$, polynomial of order 3

$$P_3(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 =$$

$$= 1 + \frac{-1/2}{1}x + \frac{-1/4}{2}x^2 + \frac{-3/8}{6}x^3$$

$$= 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$$

Question 7

Only write your final answer to the question in the box below.

4p Solve the initial value problem

$$\begin{cases} \frac{dy}{dx} = \frac{y}{x} + x\sin x \\ y(\pi) = 0 \end{cases}$$

$y =$ $-x\cos(x) - x$

Question 8

3p Find the polar form for $\frac{z}{w}$ in case $z = \sqrt{3} + i$ and $w = 1 + \sqrt{3}\,i$
Choose from the alternatives below.

- ○ A) $2e^{-\frac{\pi}{6}i}$
- ○ B) $\frac{1}{2}e^{-\frac{\pi}{6}i}$
- ○ C) $2e^{\frac{\pi}{6}i}$
- ○ D) $\frac{1}{2}e^{\frac{\pi}{6}i}$
- ○ E) $4e^{-\frac{\pi}{6}i}$
- ○ F) $e^{\frac{\pi}{6}i}$
- ● G) $e^{-\frac{\pi}{6}i}$
- ○ H) $4e^{\frac{\pi}{6}i}$

Question 9

3p Find all solutions in $\mathbb{C}$ of the equation $z^4 - 4 = 0$

$z^4 - 4 = 0 \iff (z^2-2)(z^2+2) = 0 \iff z^2 = 2$ or $z^2 = -2$

$z_1 = \sqrt{2}$, $z_2 = -\sqrt{2}$, $z_3 = i\sqrt{2}$ and $z_4 = -i\sqrt{2}$

Alternative: $z^4 - 4 = 0 \iff z^4 = 4 \iff z = \sqrt[4]{4}\, e^{\frac{k \cdot 2\pi i}{4}}$

$z_1 = \sqrt[4]{4}\, e^{0} = \sqrt{2}$, $z_2 = \sqrt[4]{4}\, e^{\frac{2\pi i}{4}} = \sqrt{2}\, i$,

$z_3 = \sqrt[4]{4}\, e^{\frac{2 \cdot 2\pi i}{4}} = -\sqrt{2}$ and $z_4 = \sqrt[4]{4}\, e^{\frac{3 \cdot 2\pi i}{4}} = -\sqrt{2}\, i$

Question 10

2p Which function is a particular solution to $y'' + y' = x$

- ○ A) $y = x$
- ○ B) $y = \frac{1}{2}x^2$
- ○ C) $y = x - e^{-x}$
- ○ D) $y = \frac{1}{2}x^2 - e^{-x}$
- ○ E) $y = 1 - e^{-x} + x$
- ● F) $y = 1 - x + \frac{1}{2}x^2$
- ○ G) $y = 1 + x - \frac{1}{2}x^2$
- ○ H) $y = x - \frac{1}{2}x^2$

Question 11

6p Determine the unique solution to the problem

$y'' + 5y' + 4y = 8\sin x + 2\cos x$

$y(0) = 0$

$y'(0) = 0$

The equation $r^2 + 5r + 4 = (r+4)(r+1) = 0$ gives us

$Y_{hom}(x) = c_1 e^{-4x} + c_2 e^{-x}$. Next choose a trial

$Y_{part}(x) = A\sin(x) + B\cos(x)$

$Y'_{part}(x) = A\cos(x) - B\sin(x)$ and substitute in

$Y''_{part}(x) = -A\sin(x) - B\cos(x)$ inhomogeneous diff. eq.

$-A\sin(x) - B\cos(x) + 5(A\cos(x) - B\sin(x)) + 4(A\sin(x) + B\cos(x)) = 8\sin(x) + 2\cos(x)$

$(-A - 5B + 4A)\sin(x) + (-B + 5A + 4B)\cos(x) = 8\sin(x) + 2\cos(x)$

$(3A - 5B)\sin(x) + (5A + 3B)\cos(x) = 8\sin(x) + 2\cos(x)$

Solve $\begin{cases} 3A - 5B = 8 \\ 5A + 3B = 2 \end{cases} \Leftrightarrow \begin{cases} 9A - 15B = 24 \\ 25A + 15B = 10 \end{cases} \Leftrightarrow \begin{cases} 34A = 34 \to A = 1 \\ \text{and } 9 - 15B = 24 \end{cases}$

so $B = -1$

General solution $y(x) = y_{hom}(x) + y_{part}(x)$, so

$y(x) = c_1 e^{-4x} + c_2 e^{-x} + \sin(x) - \cos(x)$ with

$y'(x) = -4c_1 e^{-4x} - c_2 e^{-x} + \cos(x) + \sin(x)$

$y(0) = 0 \rightarrow c_1 + c_2 + 0 - 1 = 0$

$y'(0) = 0 \rightarrow -4c_1 - c_2 + 1 + 0 = 0$

Solve $\begin{cases} c_1 + c_2 = 1 \\ -4c_1 - c_2 = -1 \end{cases} \Leftrightarrow \begin{cases} -3c_1 = 0 \rightarrow c_1 = 0 \text{ and} \\ 0 + c_2 = 1 \rightarrow c_2 = 1 \end{cases}$

Unique solution : $y(x) = e^{-x} + \sin(x) - \cos(x)$

Extra writing space

Clearly refer to this section if you use it.