Course : **Calculus 1 B**

Date : January 11, 2018
Time : 13.45 - 15.45

Motivate all answers and calculations.
The use of electronic devices is not permitted.

[2p] **1.** Determine dy/dx in case

$$y = \int_{2x}^{3x} \ln(t)\, dt$$

[4p] **2.** Determine a region in the $x-y$ plane whose area is given by

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(1+\frac{2k}{n}\right)^4 \cdot \frac{1}{n}$$

and evaluate this limit.

(Hint: Find a function $f:[0,1]\to\mathbb{R}$ for which a Riemann sum is given by $\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^4 \cdot \frac{1}{n}$.)

[2p] **3.** a) Compute

$$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\, dx$$

[3p] b) Determine

$$\int 2x\, \tan^{-1}(x)\, dx$$

by using integration by parts.

[3p] c) Compute

$$\int_e^{\infty} \frac{1}{x\,(\ln(x))^3}\, dx$$

[2p] **4.** a) Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{x^n}{2^n}$$

and within this interval the sum of the series as a function of x.

[2p] b) Find, using **4.** a), as a function of x the sum of the series

$$\sum_{n=1}^{\infty} \frac{n\,x^{n-1}}{2^n}.$$

P.T.O.

[4p] **5.** Solve the initial value problem

$$\begin{cases} (x^2+1)\,\frac{dy}{dx} = x\,y \\ y(0) = 1 \end{cases}$$

[2p] **6.** a) Find the real part of $(1+i)^{200}$.

[2p] b) Find all solutions in $\mathbb{C}$ of the equation

$$z^4 + z^3 + z^2 = 0$$

[2p] c) Find the polar form for $z = \dfrac{w_1}{w_2}$ where

$$w_1 = 2\sqrt{3} - 2i \text{ and } w_2 = -1 + i$$

[6p] **7.** Find the real-valued function y which solves

$$\begin{cases} y'' - 2y' + y = x \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$$

[2p] **8.** A *learning curve* is the graph of a function $P(t)$, the performance of someone learning a skill as a function of the training time t. At the start of the training there is a complete lack of skill, so at that moment is $P = 0$. The maximum level of performance of which the learner is capable is 5 units.
This is modelled in a differential equation

$$\frac{dP}{dt} = 2\,(5 - P).$$

Make a sketch of a possible solution $P(t)$ of this differential equation (with $t \geq 0$ in hours). Do **not** attempt to solve the differential equation.

The End.

Total: 36 points