1.

$$y = \int_0^{16x^4} \sqrt{t}e^t dt - \int_0^{x^4} \sqrt{t}e^t dt$$

$$\frac{dy}{dx} = \sqrt{16x^4}\, e^{16x^4} \cdot 64x^3 - \sqrt{x^4}\, e^{x^4} \cdot 4x^3$$

$$\frac{dy}{dx} = 256x^5 e^{16x^4} - 4x^5 e^{x^4}$$

2. **a)**

Sketch quarter of circle with radius 3 and midpoint $(0, 1)$

b)

Region consists of rectangle plus quarter of disk

Area rectangle = 3, area disk = 9π

Answer: $3 + \frac{9}{4}\pi$

3. **a)**

Antiderivative by using substitution $u = \ln(x)$

$$\int \frac{\ln(x)}{x} dx = \frac{1}{2} \ln^2(x)$$

Answer: $\frac{1}{2} \ln^2(9) - \frac{1}{2} \ln^2(4)$

b)

Substitution $u = \sqrt{1 + x^2}$

$u^2 = 1 + x^2 \implies 2u\frac{du}{dx} = 2x \implies u\, du = x\, dx$

Transform integral $\int u \cdot u\, du = \frac{1}{3}u^3$

Answer: $\frac{1}{3}\left(\sqrt{1 + x^2}\right)^3 + c$ (constant 1/2p)

c)

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx = \lim_{b\to\infty}\int_1^b \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$$

$$\int \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx \quad : \quad \text{substitution } u = \sqrt{x}, \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$\text{Transform } \int 2e^{-u}du = -2e^{-u} = -2e^{-\sqrt{x}}$$

$$\lim_{b\to\infty}\left(-2e^{-\sqrt{b}} - -2e^{-\sqrt{1}}\right) = 0 + 2e^{-1} = \frac{2}{e}$$

4.

Maclaurin series $e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + ...$

Substitute $t = -x^2$

Maclaurin series $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} \pm ...$

Maclaurin series $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \mp ...$

Answer: $1 + x - x^2 - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!}...$

$$= \sum_{k=0}^{\infty} \frac{x^{2k}}{k!} + \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

5. a)

Picture with x-axis, y-axis, Cartesian plane

Calculating slopes for at least 5 points

Drawing short line segments of slope $y - 2x$

b)

$y' - y = -2x \quad , \quad P(x) = -1 \quad , \quad Q(x) = -2x$

Integrating factor $e^{\int -1 dx} = e^{-x}$

Multiplying $\implies \frac{d}{dx}\left(e^{-x}y\right) = -2xe^{-x}$

$e^{-x}y = \int -2xe^{-x}dx = 2xe^{-x} + 2e^{-x} + c$

$y = 2x + 2 + ce^{x}$

$y(1) = 0 \implies c = \frac{-4}{e}$, answer: $y = 2x + 2 - 4e^{x-1}$

Alternative: $y' - y = 0 \implies y_H = c \cdot e^{x}$

Try $y_P = ax + b \implies y_P = 2x + 2$

$y = ce^{x} + 2x + 2$

6. a)

$1 + i = \sqrt{2}e^{i\frac{\pi}{4}}$

$(1+i)^{20} = \left(\sqrt{2}\right)^{20} \cdot e^{i\frac{\pi}{4}\cdot 20}$

$(1+i)^{20} = 1024 \cdot e^{i5\pi}$

Alternative: $(1+i)^2 = 2i$

$(1+i)^{20} = (2i)^{10}$

$= 1024e^{i\pi}$

b)

Quadratic formula: $z_{1,2} = \frac{2 \pm \sqrt{-4}}{4}$

$z_1 = \frac{1}{2} + 12i \qquad z_2 = \frac{1}{2} - \frac{1}{2}i$

c)

Picture of complex plane, points 2 and $2i$

Geometric solution graphed as perpendicular midline

or algebraic solution graphed as $\{z \mid \mathrm{Re}(z) = \mathrm{Im}(z)\}$

7. **a)**

$$y_P = \frac{1}{2}x^2 - x \implies y'_P = x - 1 \implies y''_P = 1$$

Verifying by substitution

b)

$$y'' + y' = 0 \implies r^2 + r = 0 \text{ (characteristic equation)}$$

$$r = 0 \text{ or } r = -1 \implies y_H = c_1 e^{0x} + c_2^{-x} = c_1 + c_2 e^{-x}$$

$$y = y_H + y_P = c_1 + c_2 e^{-x} + \frac{1}{2}x^2 - x$$

$$y' = -c_2 e^{-x} + x - 1$$

$$\begin{cases} y(0) = 0 & \implies & c_1 + c_2 = 0 \\ y'(0) = 0 & \implies & -c_2 - 1 = 0 \end{cases} \implies \begin{cases} c_1 = 1 \\ c_2 = -1 \end{cases}$$

Answer: $y = 1 - e^{-x} + \frac{1}{2}x^2 - x$