Course : **Mathematics B2 (Newton)**

Date : February 2, 2018
Time : 13.45 - 15.45

Motivate all answers and calculations.
The use of electronic devices is not permitted.

[3p] **1**. Determine dy/dx in case

$$y = \int_{x^4}^{16x^4} \sqrt{t}\, e^t \, dt$$

2. Consider the integral

$$I = \int_{-3}^{0} f(x)dx \quad \text{where} \quad f(x) = 1 + \sqrt{9 - x^2}$$

[1p] a) Sketch the graph of the integrand f on the interval $[-3, 0]$.

[2p] b) Evaluate the integral I by interpreting it in terms of the area under the graph of f on the interval $[-3, 0]$.

[3p] **3**. a) Compute

$$\int_4^9 \frac{\ln(x)}{x}\, dx$$

[3p] b) Determine

$$\int x\sqrt{1 + x^2}\, dx$$

[3p] c) Compute in case of convergence

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

[3p] **4**. Find the MacLaurin series for the function

$$f(x) = e^{-x^2} + \sin(x)$$

P.T.O.

[3p] **5**. a) Sketch the slope field of the differential equation

$$y' = y - 2x$$

[3p] b) Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = y - 2x \\ y(1) = 0 \end{cases}$$

[2p] **6**. a) Find and expression in the form $re^{i\theta}$ for $(1+i)^{20}$.

[2p] b) Find all solutions in $\mathbb{C}$ of the equation

$$2z^2 - 2z + 1 = 0$$

[2p] c) Graph the points $z = x + iy$ that satisfy $|z - 2i| = |z - 2|$.

[2p] **7**. a) Verify that

$$y_p = \frac{1}{2}x^2 - x$$

is a particular solution to

$$y'' + y = x$$

[4p] b) Find the real-valued function y which solves

$$\begin{cases} y'' + y' = x \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

Total: 36 points