Course : **Mathematics B2 (Newton)**

Date   : January 12, 2018
Time   : 13.45 - 15.45

Motivate all answers and calculations.
The use of electronic devices is not permitted.

1.   Determine dy/dx in case

[1.5p]   a)
$$y = \int_{-x}^{0} \sin(t^2)\, dt$$

[1.5p]   b)
$$y = x \int_{0}^{x} e^{-t^2}\, dt$$

[3p] **2.** a) Compute
$$\int_{e}^{e^2} \frac{1}{x\sqrt{\ln(x)}}\, dx$$

[3p]   b) Determine
$$\int x^2 e^{-x}\, dx$$

[3p]   c) Compute
$$\int_{0}^{\infty} xe^{-x^2}\, dx$$

3.   Find

[3p]   a) the interval of convergence of
$$\sum_{n=0}^{\infty} (\ln(x) + 1)^n$$

and within this interval the sum of the series as a function of x.

[3p]   b) the Taylor polynomial of order 3 generated by $f(x) = \sqrt{1 + x^2}$ at $x = 0$.

P.T.O.

[3p] **4.** Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dx} + xy = x \\ y(0) = -6 \end{cases}$$

[3p] **5.** The decay of radioactive Radon is described by the equation

$$\frac{dy}{dt} = -0.18y$$

with time t measured in days.

Calculate the half-life time of Radon, i.e., the time during which half the nuclei has decayed.

[3p] **6.** a) Find an expression in the form $re^{i\theta}$ for $z \cdot w$ and z/w in case

$$z = 2\sqrt{3} - 2i \quad ; \quad w = -1 + i$$

[3p] b) Find all solutions in $\mathbb{C}$ of the equation

$$z^3 + 1 = 0$$

[6p] **7.** Find the real-valued function y which solves

$$\begin{cases} y'' + 2y' + 5y = 20 \\ y(0) = 4 \\ y'(0) = 4 \end{cases}$$

Total: 36 points