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UNIVERSITY OF TWENTE.

Tag : Calc1B.23-24.Exam[01-C]
Course : **Calculus 1B**
Date : Friday January 12th, 2024
Time : 13:45 – 15:45

Motivate all your answers.

Simplify all your answers as much as possible.

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored. If you need more space, you can write in the frame provided at the end of the answer form; clearly refer to this space in the original answer. At the end of the exam, you only hand in your answer form.

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1. [2 pt] *Only write your final answer in the frame on the answer form.*

   Compute the **area** of the region bounded by the x-axis, the lines $x = 0$ and $x = 1$, and the graph of the function f given by

   $$f(x) = 5x(x-1)\,.$$

   (Hint: First sketch the graph of f.)

2. [2 pt] *Only write your final answer in the frame on the answer form.*

   Compute $f'(x)$ when $f(x)$ is given by

   $$f(x) = \int_1^{x^3} \frac{1}{1+t^3e^{4t}}\,dt\,.$$

3. [4 pt] Evaluate the following integral:

   $$\int \frac{1+\sin\left(1+\sqrt{x}\,\right)}{\sqrt{x}}\,dx\,.$$

4. [5 pt] Evaluate the following integral:

   $$\int_1^{\infty} \frac{\ln x}{x^2}\,dx\,.$$

5. [4 pt] *Only write your final answers to (a) and (b) in the frames on the answer form.*

   Consider the following power series:

   $$\sum_{n=0}^{\infty} 5^{-n}x^{3n}\,.$$

   a. [2pt] Compute the radius of convergence of this series.

   b. [2pt] Compute the sum of this series (within its radius of convergence).

Continue on the next page.

6. [3 pt] Given is the function

$$f(x) = 2x\sqrt{x}\,.$$

Determine the Taylor polynomial of order 3 generated by f at $x = 1$.

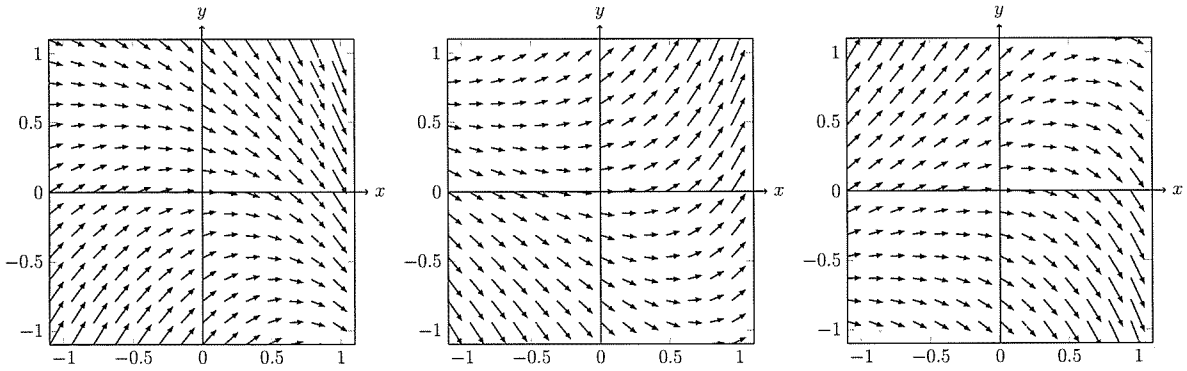
7. [6 pt] a. [4pt] Find the general solution to the differential equation

$$y' = y + e^x + 1\,.$$

b. [2pt] *Only write your final answer in the frame on the answer form.*
Consider the differential equations (I), (II) and (III) below:

(I) $y' = y + e^x - 1$
(II) $y' = y - e^x + 1$
(III) $y' = -y - e^x + 1$

For each of these differential equations, determine which of the slope fields (A, B or C) in the figure below corresponds to the equation.



(A) (B) (C)

8. [1 pt] *Only write your final answer in the frame on the answer form.*
Let $z = a+ib$ with $a, b \in \mathbb{R}$. Exactly one of the expressions below equals $\dfrac{z^2 - (\bar{z})^2}{4i}$.
Which one?

A) ab
B) $-ab$
C) a^2b^2
D) $-a^2b^2$
E) $a - b$
F) $b - a$
G) $a^2 - b^2$
H) $b^2 - a^2$

9. [3 pt] *Only write your final answer in the frame on the answer form.*
Determine all solutions (in $\mathbb{C}$) of the equation $(z + 2i)^2 = -2 + 2\sqrt{3}\,i$.
Write each solution in the form $z = a + bi$ with $a, b \in \mathbb{R}$.

10. [6 pt] Solve the following initial value problem:

$$y'' + 2y' - 3y = 5\cos x\,, \quad y(0) = \tfrac{1}{2}\,, \quad y'(0) = 0\,.$$

The End.

Total: 36 points