Kenmerk : AM2017/DMMP/014/ha

Course : **Mathematics A (Euclid)**

Date : September 22, 2017
Time : 13.45 – 14.45 hrs

Motivate all your answers.
The use of electronic devices is not allowed.

1. [3 pt]
   Let, for $k \in \{1, 2, 3, \ldots, 50\}$, the sets $A_k \subseteq \mathbb{R}$ be given by:
   $$A_k = (0, 50 + k) \cap [k, 100].$$
   (a) [1 pt] Write A_k as one interval.
   (b) [2 pt] Determine
   $$\bigcup_{k=1}^{50} A_k \quad \text{and} \quad \bigcap_{k=1}^{50} A_k.$$

2. [3 pt]
   Let A, B and C be sets in a universe $\mathcal{U}$.
   Determine a quantified statement for the following (i.e. a statement only using one or more of the following symbols: $\forall$, $\exists$, x, A, B, (,), [,], $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$, $\in$, $\notin$, $\neg$)
   $$(B \cap C) \subseteq \overline{A}.$$

3. (a) [2 pt] Let $a, b \in \mathbb{Z}$, with a even and b odd. Use the definitions of *even* and *odd* to prove that $a - b$ is odd.
   (b) [3 pt] Prove with mathematical induction that for all $n \in \mathbb{N}$,
   $$\sum_{i=1}^{n} (-1)^i \cdot i^2 = \frac{(-1)^n \cdot n(n+1)}{2}.$$

4. In this exercise your answers must be numbers; if your answer contains binomial coefficients or factorials, like $\binom{8}{3}$ or $8!$, you must expand these and compute the outcome.
   Consider six different kinds of fruit and five different kinds of candy.
   (a) [1.5 pt] In how many ways can one choose a collection of three different kinds of fruit?
   (b) [1.5 pt] If there are three boys and two girls, in how many ways can one give each boy one kind of fruit and each girl one kind of candy?

Total: 14 points