Exercise 5

(a) (3 points) Consider the following preferences and show that *no* positional voting rule satisfies the Condorcet principle.

3 voters: $A \succ B \succ C$
2 voters: $B \succ C \succ A$
1 voter: $B \succ A \succ C$
1 voter: $C \succ A \succ B$

Solution: A is the Condorcet winner, as A beats B 4:3 by majority, and A beats C 4:3 by majority. But any positional scoring rule makes B win, as $s_1 \geq s_2 \geq s_3$: The scores of A, B, and C are:

$$A : 3s_1 + 2s_2 + 2s_3$$

$$B : 3s_1 + 3s_2 + 1s_3$$

$$C : 1s_1 + 2s_2 + 4s_3$$

(b) (4 points) Argue that any resolute voting rule that is neutral, is also non-imposing. (Recall that neutrality means that F is insensitive towards a renumbering of the alternatives, $F(\pi(b_1), \ldots, \pi(b_n)) = \pi(F(b_1, \ldots, b_n))$).

(c) (5 points) Consider the following version of the card experiment as discussed in class. There are two decks, one with 8 red cards and 2 black cards, and one with 2 red and 8 black cards. Let us call the first deck the red, and the second the black deck. Any of the two decks is chosen with probability $1/2$. Once that has been done, there are 3 players who get to see, each individually, one random card of the deck. In order to decide that the deck was black, the players must unanimously vote black. In all other cases, the result is red. Let us assume that players are non-strategic (i.e., follow their private signals).

(1) What is the probability that the players decide black but the deck is red?

(2) What is the probability that the players decide black?

(3) Suppose the players have decided black. What is the probability that the deck is really black?

(4) Briefly discuss: When a player receives a red signal, should he actually be truthful and follow the signal?