Partial answers to the exam of Web Science (201500025) Exam resit Part 1 January 31, 2018, 8:45-11:45 hrs. In case you have any questions, please contact: M.deGraaf@utwente.nl

Question 3

(a) See Section 3.1 of the book: "Networks Crowds and Markets"
(b) See Section 3.2 subsection " Strong Triadic Closure Property"
(c) See Section 3.2 subsection "Local Bridges and Weak Ties" Figure 3.6
(d) A: satisfies STC, two strong edges to B and D and an edge {B,D}; B: satisfies, two strong edges to C and A and an edge {A,C}; C : does **not** satisfy STC, two strong edges to C and B, E, but no edge {B,E}; D satisfies STC, two strong edges to A and E, and an edge {A,E}; E does **not** satisfy STC: two strong edges to C and D, but no edge {C,D}.
(e) Add edges: {C,D} and {B,E} with each of them with possible labels {w: weak, s: strong}

Question 4

(a) Bayes rule: see equation 16.4
(b) If we observe H, we will choose A: Accept. Motivation: we have to show that P(G|H) > ½ (which means that given the observation of H, the probability that we are in G has increased). Bayes rule can be written as: $P(G|H) = \frac{P(G)P(H|G)}{P(H|G)P(G)+P(H|\bar{G})P(\bar{G})}$. In this specific example we have $P(G) = P(\bar{G}) = 1/2$, so after division through this number, the equation simplifies to: $P(G|H) = \frac{P(H|G)}{P(H|G)+P(H|\bar{G})}$=3/5, after substitution of P(H|G)=3/5, $P(H|\bar{G})$ =2/5. As this number exceeds the a-priori probability of ½, we choose A.
(c) The two people before choose A, which means they have observed an H signal. I observe a L signal. If P(G|HHL)>1/2, I will choose A, otherwise I will choose L. Let's calculate: $P(G|HHL) = \frac{P(G)P(HHL|G)}{P(HHL|G)P(G)+P(HHL|\bar{G})P(\bar{G})}$, in this specific example we have $P(G) = P(\bar{G}) = 1/2$, so after division through this number, the equation simplifies to: $P(G|HHL) = \frac{P(HHL|G)}{P(HHL|G)+P(HHL|\bar{G})}$. After substitution of P(H|G)=3/5, $P(H|\bar{G})$ =2/5 we find: $\frac{\frac{3}{5}\cdot\frac{3}{5}\cdot\frac{2}{5}}{\frac{3}{5}\cdot\frac{3}{5}\cdot\frac{2}{5}+\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{3}{5}} = \frac{18}{18+12} = \frac{3}{5} > \frac{1}{2}$. As this exceeds the a-priority probability we choose A.
(d) This shows that after two times a choice of A, we are in an A-cascade. This means that we already have an A-cascade after two consecutive H-signals. The probability that the state is $\bar{G}$ given that we have observed two consecutive H-signals follows from Bayes' rule: $P(\bar{G}|HH) = \frac{P(\bar{G})P(HH|\bar{G})}{P(HH|\bar{G})P(\bar{G})+P(HH|G)P(G)}$. In this specific example we have $P(G) = P(\bar{G}) = 1/2$, so after division through this number, the equation simplifies to: $P(\bar{G}|HH) = P(HH|\bar{G})P(\bar{G}) + P(HH|G)P(G) = \frac{4/25}{4/25+9/25}$ = 4/13.

Note: it is possible to interpret the question in another way, in the sense that : what is the probability that an incorrect cascade starts. Translated as 'what is the probability of two times an incorrect signal': P(incorrect cascade) = $P(HH|\bar{G})P(\bar{G}) + P(LL|G)P(G) = 4/25$.

BONUS:

After simplification we have:

$P(G|S) = \frac{P(G)P(S|G)}{P(S|G)P(G)+P(S|\bar{G})P(\bar{G})}$. In this specific example we have $P(G) = P(\bar{G})$. This means both values must be equal to ½. So after division through this number, the equation simplifies to: $P(G|S) = \frac{P(S|G)}{P(S|G)+P(S|\bar{G})}$. Suppose $P(S|\bar{G}) < P(S|G)$, so the denominator is smaller than 2P(S|G). this means the fraction is strictly ½ = P(G). Interpretation: if we are equally likely in a 'good' state of the world , as in a 'bad' state of the world, and the probability that a signal occurs in a 'bad' state of the world is strictly smaller than the probability that the same signal occurs in a 'good' state of the world, then when the signal occurs, we are more likely to be in the 'good' state of the world.

Question 5.

(a) See the book, Section 17.1.
(b) Is not a question (mistake in the exam☹)
(c) See section 17.4 we have:

$$g(z) = \begin{cases} r^{-1}\left(\frac{p^*}{f(z)}\right), when\ \frac{p^*}{f(z)} \leq r(0), \\ 0, otherwise \end{cases}$$

With $r(x) = 1 - x$, we have $r^{-1}(x) = 1 - x$. With $f(z) = \sqrt{z}, and\ r(0) = 1$ it follows:

$$g(z) = \begin{cases} 1 - \frac{p^*}{\sqrt{z}}, when\ p^* \leq \sqrt{z}, \\ 0, otherwise \end{cases}$$

(d) We have equilibrium if r(z)f(z)= p*. With z=1/9, we find (1-$\sqrt{z}$)$z = \frac{2}{3}$ ⋅ 1/9= 2/27.\
(e) Non-stable. See book, Figure 17.5.

Question 6.

(a) See Section 19.2, subsection "networked coordination game" .
(b) Nodes 1,5 and 10 adopt A. Round 1: only nodes 4, 6 adopt A. Round 2: Only node 2 adopts A. Then the process stops. Nodes {1,2,4,5,6, 10} eventually adopt A.
(c) Clusters {1,4,5,6,10}, {3,7,8,9,12} and {11,13,14,15,16} each have density > ½.
(d) Example : 6, 7, 11 (we need at least one node from each of the clusters identified above)
(e) No. If we have only two nodes then we have at best a node from two of the clusters identified under c), then the remaining network contains a cluster of density >1/2, so we cannot have complete cascade.
(f) See Section 19.3
(g) The proof of Section 16, Appendix B shows that the size of the interface decreases with at least one for each node adopting A. Given m nodes with degree k, the size of the interface is at bounded from above by m*k. So at most m*k nodes can adopt A.