Graph Theory (191520751)

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Motivate your answers.
All graphs are simple.

1. Are the following sequences degree sequences of simple graphs?
   a) $(5,5,5,3,2,2,1,1)$ **b)** $(5,5,5,4,2,1,1,1)$

2. Show that $\kappa = \kappa'$ holds for 3-regular graphs.
   (Hint: Case analysis for vertex connectivity $\kappa = 0, ..., 3$.)

3. Sketch a proof of $\tau(K_n) = n^{n-2}$. (τ = number of spanning trees.)

4. Let $G = (V, E)$ be a connected graph with $2k$ nodes of odd degree. Show: There are k edge disjoint trails $T_1, ..., T_k$ with $E = E(T_1) \cup ... \cup E(T_k)$.

5. State Tutte's Theorem on perfect matchings.
   Derive a min-max formula for the size of a maximum matching in a graph G.
   (No proof required, but you can earn 3 extra points for providing one.)

6. Let G be a simple k-regular graph with $\kappa(G) = 1$. Show that $\chi'(G) = k + 1$.

7. Prove: $\chi(G) + \chi(G^c) \leq \nu(G) + 1$. ($\nu(G)$ = number of vertices in G.)
   (Hint: Consider a smallest counterexample and argue that both G and its complement G^c must be critical.)

Points: 36+4 = 40

1: 5	2: 5	3: 5	4: 5	5: 5	6: 5	7: 6