Graph Theory (191520751) january 27, 2015, 13.45 – 16.45

Motivate your answers. All graphs are simple.

- 1. Are the following sequences degree sequences of simple graphs?
 - a) (5,5,5,3,2,2,1,1)
- **b**) (5, 5, 5, 4, 2, 1, 1, 1)
- 2. Show that $\kappa = \kappa'$ holds for 3-regular graphs. (Hint: Case analysis for vertex connectivity $\kappa = 0, ..., 3$.)
- 3. Sketch a proof of $\tau(K_n) = n^{n-2}$. ($\tau = \text{number of spanning trees.}$)
- 4. Let G = (V, E) be a connected graph with 2k nodes of odd degree. Show: There are k edge disjoint trails $T_1, ..., T_k$ with $E = E(T_1) \cup ... \cup E(T_k)$.
- 5. State Tutte's Theorem on perfect matchings.

 Derive a min-max formula for the size of a maximum matching in a graph G.

 (No proof required, but you can earn 3 extra points for providing one.)
- 6. Let G be a simple k-regular graph with $\kappa(G) = 1$. Show that $\chi'(G) = k + 1$.
- 7. Prove: $\chi(G) + \chi(G^c) \leq \nu(G) + 1$. $(\nu(G)) = \text{number of vertices in } G$.) (Hint: Consider a smallest counterexample and argue that both G and its complement G^c must be critical.)

Points: 36+4 = 40

1: 5 | 2: 5 | 3: 5 | 4: 5 | 5: 5 | 6: 5 | 7: 6

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