Exam Limits to Computing (202600265)

Wednesday, October 28, 2026, 13:45 – 16:45

- You can bring five sheets of A4 paper with arbitrary content to the exam.
- Electronic devices of any kind, books, ... are not allowed.
- This exam consists of five problems.
- Please start a new page for each problem.
- The total number of points is 70. Sufficient for passing are 35 points.

Exercise 1	Exercise 2	Exercise 3	Exercise 4	Exercise 5
a: 9 points	8 points	a: 7 points	10 points	a: 2 points
b: 9 points		b: 9 points		b: 2 points
c: 2 points				c: 2 points
d: 6 points				d: 2 points
				e: 2 points
26 points	8 points	16 points	10 points	10 points

1. Computability

Let

$$\textsc{BoundedDom} = \{g \in \mathcal{G} \mid |\mathrm{dom}(\varphi_g)| \leq g\}.$$

(a) *(9 points)* Is $\textsc{BoundedDom} \in \mathsf{REC}$?

(b) *(9 points)* Is $\textsc{BoundedDom} \in \mathsf{co\text{-}RE}$?

(c) *(2 points)* Is $\textsc{BoundedDom} \in \mathsf{RE}$?

(d) *(6 points)* Is $\textsc{BoundedDom}$ an index set?

2. NP-Completeness 1

A Hamiltonian cycle of an undirected graph $G = (V, E)$ is a simple cycle of G that contains every vertex of G exactly once. Let

$$\textsc{HamCycle} = \{G \mid G \text{ contains a Hamiltonian cycle}\}.$$

HAMCYCLE is NP-complete.

A *half cycle* in an undirected graph $G = (V, E)$ is a simple cycle in G that contains exactly $\lfloor |V|/2 \rfloor$ distinct vertices. Let

$$\textsc{HalfCycle} = \{G \mid G \text{ contains a half cycle}\}.$$

(8 points) Prove that HALFCYCLE is NP-complete.

3. Reductions and Complexity

We say that A is *quasi-linear-time reducible* to B (denoted by $A \leq_q B$) if there exists a constant $c \geq 0$ and a function f that is computable in time $O(n \cdot (\log n)^c)$ such that $x \in A$ if and only if $f(x) \in B$ for all x.

(a) *(7 points)* Show that $\leq_q$ is transitive.

(b) *(9 points)* Prove that there are no PSPACE-complete problems with respect to $\leq_q$. This means that there is no $L \in \mathsf{PSPACE}$ with $A \leq_q L$ for all $A \in \mathsf{PSPACE}$.

4. NP-Completeness 2

(10 points) Prove or disprove the following statement: There exists a language L such that

- L is NP-complete and
- $L \in \mathsf{DSpace}(O(\sqrt{n}))$.

5. Statements

Are the following statements true or false? Justify your answer.

(a) *(2 points)* The following holds for all sets $A, B \subseteq \mathbb{N}$: If $A \in \mathsf{REC}$ and $B \in \mathsf{RE}$, then $A \setminus B \in \mathsf{co\text{-}RE}$.

(b) *(2 points)* The following holds for all sets $A \subseteq \mathbb{N}$: $A \notin \mathsf{REC}$ if and only if $H_0 \leq A$.

(c) *(2 points)* For all $A, B \subseteq \mathbb{N}$, the following holds: If $A \cap B \in \mathsf{REC}$ and $A \cup B \in \mathsf{REC}$, then $A \in \mathsf{REC}$ and $B \in \mathsf{REC}$.

(d) *(2 points)* There exists a complexity class C with $\mathsf{co\text{-}C} \subsetneq \mathsf{C}$.

(e) *(2 points)* If $\mathsf{L} = \mathsf{NL}$, then $\mathsf{DSpace}(O(n)) = \mathsf{NSpace}(O(n))$.