Discrete Mathematics for Computer Science, October 2, 2017
Solution/Correction standard

1. (a) $$\forall_i \forall j \forall k \, [a_{ij} = a_{ik}] \quad \text{or} \quad \forall_i \forall j \in \{1, \ldots, n-1\} \, [a_{ij} = a_{i,j+1}] \, .$$ **[2 pt]**

   (b) $$\forall j \, [\exists_i (a_{ij} = 0) \wedge \exists k (a_{kj} = 1) \wedge \forall \ell (0 \leq a_{\ell j} \leq 1)] \, .$$ **[4 pt]**

   For each expression that is not logically equivalent to the ones above: 0 pt.

2.

(1)	q	Extra Premise
(2)	$p \vee r$	Premise
(3)	$\neg\neg p \vee r$	(2), L1
(4)	$\neg p \to r$	(3), L12
(5)	$p \to (\neg q \vee r)$	Premise
(6)	$\neg(\neg q \vee r) \to \neg p$	(5), L13
(7)	$\neg(\neg q \vee r) \to r$	(6),(4), R2
(8)	$\neg\neg(\neg q \vee r) \vee r$	(7), L12
(9)	$\neg q \vee (r \vee r)$	(8), L1,L4
(10)	$\neg q \vee r$	(9), L8
(11)	$\neg\neg q$	(1), L1
(12)	r	(11), R5

[6 pt]

For each forgotten Law or Rule: -1 pt.
If deduction contains a step that is not logically correct: at most 1 pt for the entire exercise.
Remark: Also R11 can be used, e.g, by first creating a T_0:
(1) $p \vee r$ (Prem); (2) $(p \vee r) \wedge T_0$ ((1),L7); (3) T_0 ((2),L3,R7); (4) $r \vee \neg r$ ((3),L8);
(5) $r \to r$ ((4),L3,L12); (6) $p \to (\neg q \vee r)$ (Prem).
Now (6),(5),(1) and R11 imply $(\neg q \vee r) \vee r$. Then applying L4, L6 and L12 respectively leads to the conclusion $q \to r$.

3. Suppose $A - C = B - C$ and $C - A = C - B$. We must show that $A = B$. **[1 pt]**
   We show that $A \subseteq B$ and $B \subseteq A$. **[1 pt]**

   (i) Proof of $A \subseteq B$.
   Let $x \in A$. We distinguish the cases $x \in C$ and $x \notin C$. **[1 pt]**
   <u>Case 1</u>: Suppose $x \in C$. Then $x \notin C - A$. So $x \notin C - B$. Then necessarily $x \in B$ (because $x \in C$ and $x \notin B$ would imply $x \in C - B$). **[1 pt]**
   <u>Case 2</u>: Suppose $x \notin C$. Then $x \in A - C$. So $x \in B - C$. So again $x \in B$. **[1 pt]**
   From Case 1 and Case 2 we conclude $A \subseteq B$.

   (ii) Proof of $B \subseteq A$.
   This proof is analogous to part (i), by interhanging the roles of A and B. **[1 pt]**