Kenmerk : TW2014/DWMP/032/ha

Course : **Discrete Mathematics for Computer Science**
Date : November 7, 2014
Time : 08.45-10.30 hrs

Motivate all your answers.
The use of electronic devices is not allowed.
A formula sheet is included.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. Let A and B be sets and let $f : A \to B$ be a function.
   Give quantified expressions for the following statements.
   (a) [3 pt] f is one-to-one.
   (b) [3 pt] f is onto.

2. [6 pt]
   Prove the validity of the following argument using the "Laws of Logic" and the "Rules of Inference".

$$\begin{array}{l} (p \to q) \to r \\ p \to (q \vee s) \\ \hline \therefore r \vee s \end{array}$$

3. Let A, B and C be sets in a universe $\mathcal{U}$.
   (a) [4 pt] Prove that: $(A \Delta B) \cap (B \Delta C) \subseteq A \Delta (B \cap C)$.
   (b) [2 pt] Show with a counterexample that the converse inclusion of part (a) is not necessarily true.

4. [6 pt]
   Prove with mathematical induction that for all $n \in \mathbb{N}$, $2^{3n+1} - 2$ is divisible by 7.

5. Let A, B and C be sets and let $f : A \to B$ and $g : B \to C$ be functions such that $g \circ f$ is one-to-one.
   (a) [4 pt] Prove that f is one-to-one.
   (b) [2 pt] Show with a counterexample that g is not necessarily one-to-one.

6. Let $A = \{2, 3, 8, 12, 18, 24, 36, 72\}$, $B = \{18, 24, 36\}$ and let R be the relation on A given by: xRy if and only if y is divisible by x.
   (a) [3 pt] Show that (A, R) is a poset.
   (b) [3 pt] Construct a Hasse diagram for (A, R) and determine the least upper bound and greatest lower bound of B, if they exist. Is (A, R) a lattice?

Total: 36 points