Kenmerk : TW2015/DWMP/010/ha

Course : **Discrete Mathematics for Computer Science**
Date : October 23, 2015
Time : 08.45–09.45 hrs

Motivate all your answers. The use of electronic devices is not allowed.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

1. [6 pt]
Let the sequence of numbers $a_1, a_2, a_3, \ldots$ be given by:

$a_1 = 3$, $a_2 = 11$, and for $n \geq 3$: $\quad a_n = 2a_{n-1} + 4a_{n-2}$.

Prove with mathematical induction that for all $n \in \mathbb{Z}^+$, $\quad a_n \leq \left[\frac{10}{3}\right]^n$.

2. Let A, B and C be sets and let $f : A \to B$ and $g : B \to C$ be functions such that $g \circ f$ is one-to-one.

(a) [4 pt] Prove that f is one-to-one.

(b) [2 pt] Show with a counterexample that g is not necessarily one-to-one.

3. Let $A = \{2, 3, 8, 12, 18, 24, 36, 72\}$, $B = \{18, 24, 36\}$ and let R be the relation on A given by: xRy if and only if y is divisible by x (i.e, $y = kx$ for some $k \in \mathbb{Z}$).

(a) [3 pt] Show that (A, R) is a poset.

(b) [3 pt] Construct a Hasse diagram for (A, R) and determine the least upper bound and greatest lower bound of B, if they exist. Is (A, R) a lattice?

Total: 18 points