Tag : Calc1B.21-22.SampleTest1

Course : **Calculus 1B**

Duration : 2 hours

Motivate all your open answers.

The use of electronic devices is not allowed.

The exam

There are 36 points in this exam:

Multiple Choice (7 points): Q1, Q5 and Q9.

Final Answer (6 points): Q2, Q8.

Open Answer (23 points): Q3, Q4, Q6, Q7 and Q10.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Multiple Choice
Only select the most appropriate answer out of the alternatives on the answer form.

Final answer
On the answer form, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer
You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

Continue on the next page.

1. [3 pt] Given is the function

$$f(x) = \frac{x+1}{x^3}\,.$$

Find a formula for the Riemann sum for f on the interval $[2,3]$ by dividing $[2,3]$ into n equal subintervals and using the right-hand endpoint of each subinterval to evaluate f.

Choose from the alternatives below.

A) $\displaystyle\sum_{k=1}^{n} \frac{\frac{k-1}{n}+3}{n\left(\frac{k-1}{n}+2\right)^3}$

B) $\displaystyle\sum_{k=1}^{n} \frac{\frac{k}{n}+2}{n\left(\frac{k}{n}+3\right)^3}$

C) $\displaystyle\sum_{k=1}^{n} \frac{n(k+3n-1)}{(k+2n-1)^3}$

D) $\displaystyle\sum_{k=1}^{n} \frac{\frac{k-\frac{1}{2}}{n}+3}{n\left(\frac{k-\frac{1}{2}}{n}+2\right)^3}$

E) $\displaystyle\sum_{k=1}^{n} \frac{\frac{k-1}{n}+2}{n\left(\frac{k-1}{n}+3\right)^3}$

F) $\displaystyle\sum_{k=1}^{n} \frac{n(k+2n)}{(k+3n)^3}$

G) $\displaystyle\sum_{k=1}^{n} \frac{\frac{k-\frac{1}{2}}{n}+2}{n\left(\frac{k-\frac{1}{2}}{n}+3\right)^3}$

H) $\displaystyle\sum_{k=1}^{n} \frac{\frac{k}{n}+3}{n\left(\frac{k}{n}+2\right)^3}$

2. [2 pt] Only write your final answer in the box on the answer form.

Determine

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{2-x^3}^{3} \frac{1}{1+|t|}\,\mathrm{d}t\,.$$

3. [4 pt] Compute

$$\int_{e^2}^{e^3} \frac{\ln(x)\sqrt{\ln(x)}}{x}\,\mathrm{d}x\,.$$

Simplify your answer as much as possible.

Continue on the next page.

4. [4 pt] Determine

$$\int_0^\infty x\, e^{-2x}\, \mathrm{d}x\,.$$

5. [2 pt] Compute the sum of

$$\sum_{n=0}^{\infty} 8\, \frac{x^n}{(x+1)^{2+n}} \text{ for } x > -\tfrac{1}{2}\,.$$

Choose from the alternatives below.

A) $\dfrac{8}{(x+1)^2} \cdot \dfrac{1}{1-x}$

B) $\dfrac{8}{x+1} \cdot \dfrac{1}{1-x}$

C) $\dfrac{8}{x+1} \cdot \dfrac{1}{x}$

D) $\dfrac{8}{(x+1)^2} \cdot \dfrac{1}{x}$

E) $-\dfrac{8}{(x+1)^2} \cdot \dfrac{1}{2x+1}$

F) $\dfrac{8}{(x+1)^2}$

G) $\dfrac{8}{x+1}$

H) $-\dfrac{8}{x+1} \cdot \dfrac{1}{2x+1}$

6. [3 pt] Given is the function

$$f(x) = e^{\frac{x}{2}} \cos(2x)\,.$$

Determine the Taylor polynomial of order 3 generated by f at $x = 0$.
(Hint: You could use standard Maclaurin series.)

7. [6 pt] Consider the following differential equation:

$$\sqrt{x}\, \frac{\mathrm{d}y}{\mathrm{d}x} = y + \tfrac{1}{2}\, e^{\sqrt{x}}\,.$$

a. [2pt] Show that $y = e^{2\sqrt{x}} - e^{\sqrt{x}}$ is a solution to the above differential equation.

b. [4pt] Solve the above differential equation subject to the initial condition

$$y(0) = 1\,.$$

Continue on the next page.

8. [4 pt] Only write your final answers to (a) and (b) in the boxes on the answer form.

Let $z = -1\frac{1}{2} + \frac{1}{2}\, i\, \sqrt{3}$.

a. [2pt] Express z in the form $r\, e^{i\theta}$ with $r \geq 0$ and $-\pi < \theta \leq \pi$.

b. [2pt] Use your answer to (a) to compute

$$w = \bar{z}^6\,,$$

where $\bar{z}$ denotes the complex conjugate of z.
Express w in the form $a + i\, b$ with a and b real numbers.

Simplify your answers as much as possible.

9. [2 pt] Let $z = x + i\, y$.

Determine all solutions in $\mathbb{C}$ of the equation

$$|z|^2 = 8\,\mathrm{Re}(z) - 2\,\mathrm{Im}(z) + 19\,,$$

where $\mathrm{Re}(z)$ is the real part of z and $\mathrm{Im}(z)$ is the imaginary part of z.

Choose from the alternatives below.

A) $(x-4)^2 + (y-1)^2 = 2$
B) $(x-4)^2 + (y+1)^2 = 2$
C) $(x-4)^2 + (y+1)^2 = 36$
D) $(x-4)^2 + (y-1)^2 = 36$
E) $(x+4)^2 + (y-1)^2 = 2$
F) $(x+4)^2 + (y+1)^2 = 2$
G) $(x+4)^2 + (y-1)^2 = 36$
H) $(x+4)^2 + (y+1)^2 = 36$

10. [6 pt] Determine the unique (real) solution to the following initial value problem:

$$\begin{cases} y'' + 5y' + 6y = 10\cos(x) \\ y(0) = 10 \\ y'(0) = 0 \end{cases}$$

The End.

Total: 36 points