Tag : Calc1B.21-22.Exam[01-xwv4BcA2]

Course : **Calculus 1B**

Date : Friday January 14th, 2022

Time : 13:45 – 15:45

Simplify all your answers as much as possible!

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Multiple Choice

Only select the most appropriate answer out of the alternatives on the answer form.

Final answer

On the answer form, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer

You provide a full calculation and motivation in the text frame corresponding to the question. The calculation and motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

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1. [2 pt] Given is the function
$$f(x) = e^x.$$

Find a formula for the Riemann sum for f on the interval $[1,4]$ by dividing $[1,4]$ into n equal subintervals and using the left-hand endpoint of each subinterval to evaluate f.

Choose from the alternatives below.

A) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3k}{n}}$

B) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3k}{n}+1}$

C) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3k}{n}-1}$

D) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3(k-1)}{n}}$

E) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3(k-1)}{n}+1}$

F) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3(k+1)}{n}+1}$

G) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3(k-1)}{n}-1}$

H) $\displaystyle\sum_{k=1}^{n} \frac{3}{n} e^{\frac{3(k+1)}{n}-1}$

2. [2 pt] *Only write your final answer in the box on the answer form.*

Determine
$$\frac{d}{dx} \int_2^{\frac{1}{x}} \sin\left(t^2\right) dt \,.$$

3. [4 pt] Evaluate
$$\int_1^2 \frac{1}{x\sqrt{x-1}}\, dx \,.$$

4. [4 pt] Evaluate
$$\int 4x^3 e^{x^2}\, dx \,.$$

5. [4 pt] *Only write your final answers to (a) and (b) in the boxes on the answer form.*

Consider the following power series:
$$\sum_{n=1}^{\infty} 2\left(\frac{1-3x}{4}\right)^n \,.$$

a. [2pt] Compute the interval and radius of convergence of this series.

b. [2pt] Compute the sum of this series (within its interval of convergence).

6. [4 pt] Given is the function
$$f(x) = \sin^2 x \,.$$

Determine the Taylor polynomial of order 4 generated by f at $x = \frac{\pi}{2}$.

7. [5 pt] Solve the following differential equation subject to the given initial condition:
$$x\frac{dy}{dx} = 2\sqrt{x} + 3y \,, \quad y(1) = 0 \,.$$

8. [6 pt] *Only write your final answers to (a), (b) and (c) in the boxes on the answer form.*

Let $z = -1 + i$.

a. [1pt] Express z in the form $r\, e^{i\theta}$ with $r \geq 0$ and $-\pi < \theta \leq \pi$.

b. [2pt] Find the two square roots of z.

c. [3pt] Determine the real part and the imaginary part of the complex number
$$w = \frac{1}{z^6} \,.$$

(Hint: Use your answer to part a.)

9. [5 pt] Determine the unique (real) solution $y = y(x)$ to the following initial value problem:
$$y'' - 6y' + 10y = 5 \,, \quad y(0) = 0 \,, \quad y'(0) = 1 \,.$$

The End.

Total: 36 points