Test Exam ADM, Module 7, Codes 202500366 & 202001360
Algorithmic Discrete Mathematics
March 27th, 2026, 13:45–15:45

Answers to questions 1-5 need to be motivated, arguments and proofs must be complete. A cheat-sheet is provided with the exam. Other aids (including the use of electronic devices) are not allowed.

For information: The total number of points is 36. Your grade is determined as follows:

- Students registered for course code 202500366 (standalone 3EC course): $1 + 0.25x$, with x being the number of obtained points. That means you need 18 points to get a 5.5.
- **Repeat students** registered for course code 202001360 (5EC course, combined with ADS):

$$\frac{X_{\text{ADS}}}{20} + \frac{5X_{\text{ADM}}}{36} + 1,$$

where X_{ADS} and X_{ADM} are the number of points obtained in the ADS and ADM exams respectively. Note that for repeat students (registered for Course Code 202001360) Question 6(e) (indicated with a "(∗)") will not be graded.

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1. (2+3 points)
   (a) Consider $a, b, c \in \mathbb{Z}$ with $\gcd(a, b) = 1$ and $a|b^2c$. Prove or disprove that $a|c$.
   (b) Consider $a, b \in \mathbb{Z}$ with $a > b$ and $\gcd(a, b) = 1$. Prove that $\gcd(a-b, a+b) = 1$ or 2.
2. (2+3 points)
   (a) Consider a simple graph $G = (V, E)$ and two perfect matchings M_1, M_2 in G. Consider the subgraph H of G that also has V as its vertex set, and its edge set is given by the symmetric difference[1] $M_1 \triangle M_2$. Prove that, for any $v \in V$, the degree of v in H is either 0 or 2.
   (b) Prove that a tree has at most one perfect matching.
   [Hint: you may use part (a).]
3. (6 points) Consider a simple, capacitated network $G = (V, E, c)$, where V is the set of vertices, $s, t \in V$, E is the set of directed edges, and $c(e) \in \mathbb{Z}$ for all $e \in E$ are the integer edge capacities. Let $|V| = n$ and $|E| = m$. Suppose you are given a maximum (s,t)-flow f in G, and that f was computed by the Ford-Fulkerson algorithm.

   Consider now a simple, capacitated network $G' = (V, E', c')$ with $E' = E \cup \{(x, y)\}$ for some $x, y \in V$ such that $(x, y) \notin E$. Let $c'(e) = c(e), \forall e \in E$ and $c'((x, y)) = 1$.

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[1] Recall that $A \triangle B$ is defined as $(A \setminus B) \cup (B \setminus A)$.

In other words G' is identical to G, except that it has an additional directed edge of capacity one between vertices x and y.

Suggest how to compute a maximum (s,t)-flow f' in G' in $O(n+m)$ time. Briefly explain (i) why your suggested algorithm is correct (ii) why it achieves the desired running time.
(Hint: You may want to use the (s,t)-flow f as well as the residual graph for G' with respect to f.)

4. (3+2 points)

   (a) Compute the solution to the recurrence relation

$$a_n = 3a_{n-1} - 2a_{n-2} + 4 \quad (n \geq 2) \quad \text{with} \quad a_0 = 1 \text{ and } a_1 = 3\,.$$

   (b) Consider a staircase with n steps. Starting at step 0, a person can make the following moves: up by 1 step, up by 2 steps and up by 5 steps. Let a_n be the number of distinct sequences of moves (where order matters) that lead exactly to step n. Compute a_1, a_2 a_3, a_4 and a_5 and a recurrence relation for a_n for all $n \geq 6$. (You do not need to solve this recurrence relation.)

5. (5 points) Assume that Alice has published modulus $n = 51$, and exponent $e = 7$. Bob sends ciphertext $C = 3$ to Alice. You are eavesdropper Eve and you are interested in Bob's secret message M. Compute Bob's secret message M from ciphertext C. In doing that, write down all of the computational steps that you need to perform in order to obtain Bob's secret message M.

6. For each of the following claims, decide if true or false or if you would rather not give an answer. A correct answer gives $\mathbf{X}$, an incorrect answer $-\mathbf{X}$ and not giving an answer **0 points** (minimum total number of points for Question 6 is 0 points). **Instead of guessing, it may be better not giving an answer.**

   - Students registered under course code 202500366 (3EC standalone ADM course) need to answer all 5 subquestions, and each subquestion is worth $X = 2$ points.
   - **Repeat students** that are registered under course code 202001360 (5EC course, combining ADS and ADM parts), will be graded only on subquestions (a) – (d) and each exercise is worth $X = 2.5$ points.

   (a) Consider a capacitated network $G = (V, E, c)$ where $c: E \to \mathbb{Z}_{\geq 0}$ are the edge capacities. Then for any maximum s-t flow f, all edges leaving the source s must be saturated.

   True . . . . ☐
   False . . . . ☐
   I prefer to not give an answer . . . . ☐

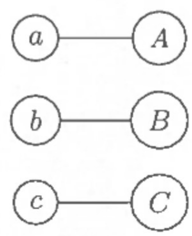
(b) Consider an undirected, simple graph $G = (V, E)$ with edge weights $w_e \geq 0$. Suppose we increase the weight of every edge in the graph by the same constant $c > 0$. Then the shortest path between any pair of vertices remains the same (only its weight will be higher than before).

True . . . . ☐
False . . . . ☐
I prefer to not give an answer . . . . ☐

(c) There exists a simple graph $G = (V, E)$, with $|V| = 9$ and $\deg(v) = 3$ for all $v \in V$.

True . . . . ☐
False . . . . ☐
I prefer to not give an answer . . . . ☐

(d) Consider the depicted matching M and corresponding preference lists. Then M is a stable matching.



$A >_a B >_a C$
$A >_b C >_b B$
$A >_c B >_c C$
$a >_A b >_A c$
$a >_B b >_B c$
$a >_C c >_C b$

[Reminder: $x >_z y$ indicates that z prefers to be matched with x over y.]

True . . . . ☐
False . . . . ☐
I prefer to not give an answer . . . . ☐

(e) (∗) Consider a capacitated network $G = (V, E, c)$, where V is the set of vertices, E is the set of directed edges, and $c : E \to \mathbb{Z}_{\geq} 0$, are the edge capacities. Then, if there are two different minimum s-t-cuts, there must also exist two different maximum flows.

True . . . . ☐
False. . . . . ☐
I prefer to not give an answer . . . . ☐
I am a repeat student (and registered under 202001360) . . . . . ☐