Exam ADS, Module 7, Codes 202001360 & 202500367

Algorithms and Data Structures

Friday March 6, 2026, 13:45 - 15:45

All answers need to be motivated! You are allowed to use a handwritten cheat sheet (A4, both sides) during the exam.

Total amount of points 90, grade: $0.1x + 1$ where x is the number of points obtained.

Repeat students: you do not need to do the starred exercise 3. Number of grade points for this exam: $0.05x$ where x is the number of points obtained (so a maximum of 4 grade points; you get a grade for the "old" ADM course component by also doing the ADM exam on March 27 for which you can get an additional 5 grade points plus 1 extra point).

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1. (20 points)

1(a). The following algorithm is a binary search on a sorted array, where // is integer division (so 5//2=2):

```
def bs(arr, target, left, right):
    if left > right:
        return -1
    if left = right
        if arr[left] == target
           return left
        else return -1

    mid = left+(right-left)//2

    if arr[mid] < target:
        return bs(arr, target, mid+1, right)
    else:
        return bs(arr, target, left, mid)
```

Let $T(n)$ be the number of comparisons where n is the size of the array. Give a recursive equation for $T(n)$. What is the value of $T(1)$?

1(b). Suppose that the number of steps of an algorithm $T(n)$ with an input of n, has the recurrence relation

$$T(n) = 3T(n/3) + n/2$$

What is the asymptotic complexity class of this algorithm?

2. (30 points)

 2(a). At a party you meet a person who claims to have an algorithm that transforms a minheap of size n into a maxheap (with the same elements), with asymptotic time complexity $\Theta(n)$. Do you believe that person? Motivate your answer.

 2(b). Give a perfectly balanced binary tree where the keys contains all the numbers 1 to 15 in such a way that if you traverse the tree in post–order fashion, you encounter the numbers in increasing order.

 What is the sequence of numbers if you traverse this tree in a pre-order way?

 2(c). Given a non-empty binary search tree with unique elements. Give an algorithm (in words or in pseudocode) that yields the biggest element smaller than the maximum.

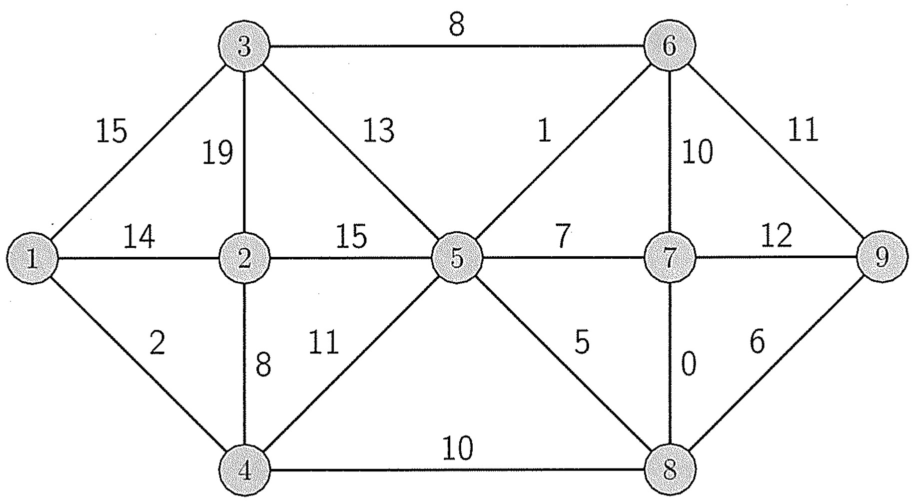
*3. **Not for repeat students!** (10 points)

Prove the following proposition using induction:

For all $h \geq 0$: a tree of height h has at most $2^h - 1$ internal nodes.

4. (10 points)

Given the following weighted graph:



Determine the minimum spanning tree using Prim's algorithm; indicated the order in which you find the edges (start at vertex 1).

5. (20 points)

Given n positive integers $a_1, \ldots, a_n$ and positive integer G. We want to determine whether there is a subsequence of $a_1, \ldots, a_n$ with sum G. Example: for $7, 3, 2, 5, 8$ and $G = 14$ the answer is "true" since $7 + 2 + 5 = 14$.

Define boolean $R(i, g)$ as:
$R(i, g)$ iff $a_1, \ldots, a_i$ has subsequence with sum g.

Note that

- $R(i, 0) = true$ for all $0 \le i \le n$
- $R(0, g) = false$ for all $g > 0$
- $R(i, g) = false$ for all $g < 0$

5(a). Which of the following recurrence relations holds (for all $1 \le i \le n$, $1 \le g \le G$):

1. $R(i, g) = max\{R(i-1, g), a_i + R(i, g)\}$
2. $R(i, g) = R(i-1, g) || R(i-1, g - a_i)$
3. $R(i, g) = min\{R(i+1, g), R(i+1, g + a_i)\}$
4. $R(i, g) = R(i-1, g) || R(i, g + a_i)$

Explain why the right answer is right.

5(b). Give an algorithm to determine whether $a_1, \ldots, a_n$ has a subsequence with sum G. The complexity may be no worse than $\Theta(Gn)$. Hint: fill an array R in the right order.