

Resit ADM, Module 7, Codes 202500366 & 202001360

Algorithmic Discrete Mathematics

June 15th, 2026, 8:45–10:45

Answers to questions 1-5 need to be motivated, arguments and proofs must be complete. A cheat-sheet is provided with the exam. Other aids (including the use of electronic devices) are not allowed.

For information: The total number of points is 36. Your grade is determined as follows:

- Students registered for course code 202500366 (standalone 3EC course): $1 + 0.25x$, with x being the number of obtained points. That means you need 18 points to get a 5.5.
- Repeat students registered for course code 202001360 (5EC course, combined with ADS):

$$\frac{X_{ADS}}{20} + \frac{5X_{ADM}}{36} + 1,$$

where X_{ADS} and X_{ADM} are the number of points obtained in the ADS and ADM exams respectively. Note that for repeat students (registered for Course Code 202001360) Question 6(e) (indicated with a "(*)") will not be graded.

1. (2+3 points)

- (a) Consider $a, b, c \in \mathbb{Z}$ with $a \mid bc$. Prove or disprove that: $a \mid b$ or $a \mid c$.
- (b) Prove Euclid's Lemma: Consider $a, b \in \mathbb{Z}$ and p prime with $p \mid ab$. Then $p \mid a$ or $p \mid b$ (or both).

2. (3+2 points)

- (a) Consider a simple graph $G_1 = (V_1, E_1)$ with average degree greater than 2. Prove that G_1 contains a cycle.
- (b) Consider a simple connected graph $G_2 = (V_2, E_2)$ with $|V_2| = |E_2|$. Prove that G_2 contains *exactly one* cycle.

3. (6 points) Consider a simple, capacitated network $G = (V, E, c)$, where V is the set of vertices, $s, t \in V$, E is the set of directed edges, and $c(e) \in \mathbb{Z}$ for all $e \in E$ are the integer edge capacities. Let $|V| = n$ and $|E| = m$. Suppose you are given a maximum (s, t) -flow f in G , and that f was computed by the Ford-Fulkerson algorithm.

Consider now a simple, capacitated network $G' = (V, E, c')$ with $c'(e) = c(e), \forall e \in E \setminus \{e^*\}$ and $c'(e^*) = c(e) + 5$. In other words G' is identical to G , except that the capacity of a specific edge e^* has increased (additively) by 5.

Suggest how to compute a maximum (s, t) -flow f' in G' in $O(n + m)$ time. Briefly explain (i) why your suggested algorithm is correct (ii) why it achieves the desired running time.

(Hint: You may want to use the (s, t) -flow f as well as the residual graph for G' with respect to f .)

4. (3+2 points)

(a) Compute the solution to the recurrence relation

$$a_n = 4a_{n-1} - 4a_{n-2} + 3^n \quad (n \geq 2) \quad \text{with} \quad a_0 = 0 \text{ and } a_1 = 1.$$

(b) Consider a $1 \times n$ strip that we want to fill using two types of tiles:

- *black* tiles of length 1 (that is, (1×1) -tiles)
- *white* tiles of length 2 (that is, (1×2) -tiles)

Let a_n be the number of distinct possible ways to fill the $1 \times n$ strip without two white tiles being placed next to each other. Compute a_1, a_2, a_3 and a_4 and a recurrence relation for a_n for all $n \geq 4$. (You do not need to solve this recurrence relation.)

5. (5 points) Assume that Alice has published modulus $n = 55$, and exponent $e = 7$. Bob sends ciphertext $C = 5$ to Alice. You are eavesdropper Eve and you are interested in Bob's secret message M . Compute Bob's secret message M from ciphertext C . In doing that, write down all of the computational steps that you need to perform in order to obtain Bob's secret message M .

6. For each of the following claims, decide if true or false or if you would rather not give an answer. A correct answer gives X , an incorrect answer $-X$ and not giving an answer 0 points (minimum total number of points for Question 6 is 0 points). **Instead of guessing, it may be better not giving an answer.**

- Students registered under course code 202500366 (3EC standalone ADM course) need to answer all 5 subquestions, and each subquestion is worth $X = 2$ points.
- Repeat students that are registered under course code 202001360 (5EC course, combining ADS and ADM parts), will be graded only on subquestions (a) – (d) and each exercise is worth $X = 2.5$ points.

(a) Consider a capacitated network $G = (V, E, c)$ where $c : E \rightarrow \mathbb{Z}_{\geq 0}$ are the edge capacities. Assume that G has a unique minimum s - t cut. Then G must also have a unique maximum flow.

True ☐

False ☒

I prefer to not give an answer ☐

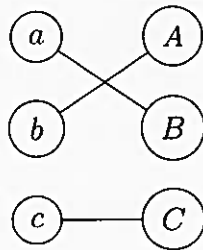
- (b) Consider an undirected, simple graph $G = (V, E)$ with edge weights $w_e \geq 0$. Suppose we multiply the weight of every edge in the graph by the same constant $c > 1$. Then the shortest path between any pair of vertices remains the same (only its weight will be higher than before).

True ☒
 False ☐
 I prefer to not give an answer ☐

- (c) A connected simple graph is Eulerian *if and only if* none of its edges is a bridge.

True ☐
 False ☒
 I prefer to not give an answer ☐

- (d) Consider the depicted matching M and corresponding preference lists. Then M is a stable matching.



$B >_a A >_a C$
 $A >_b B >_b C$
 $A >_c B >_c C$
 $a >_A b >_A c$
 $b >_B a >_B c$
 $b >_C c >_C a$

[Reminder: $x >_z y$ indicates that z prefers to be matched with x over y .]

True ☒
 False ☐
 I prefer to not give an answer ☐

- (e) (*) Consider a capacitated network $G = (V, E, c)$, where V is the set of vertices, E is the set of directed edges, and $c : E \rightarrow \mathbb{Z}_{\geq 0}$, are the edge capacities. If some edge $e \in E$ is not part of any minimum cut of G , then there exists a maximum flow f in G such that $f(e) = 0$.

True ☒
 False ☐
 I prefer to not give an answer ☐
 I am a repeat student (and registered under 202001360) ☐

