

Exam 1 ADS, Module 7, Codes 202001360 & 202001364

Algorithms and Data Structures

Monday March 31, 2025, 8:45 - 9:45

All answers need to be motivated! You are allowed to use a handwritten cheat sheet (A4, both sides) during the exam.

For information: this exam is the ADS part. The entire exam consists of two parts:

Algorithms & Data Structures (ADS)	1h	(30 points) today's test
Discrete Mathematics (DM)	2h	(60 points) last Friday's test

The total is $30+60=90$ points. Your grade is $1 + 0.1x$, x being the number of points, rounded to one digit. That means you need 45 points to get a 5.5.

1. (10 points)

(a) Consider this sorting algorithm that sorts an array *arr* of integers:

```
def Sort(arr):
    for i in range(1, len(arr)):
        key = arr[i]
        j = i - 1
        while j >= 0 and arr[j] > key:
            arr[j + 1] = arr[j]
            j = j - 1
        arr[j + 1] = key
```

Give the asymptotic time complexity of this algorithm, expressed in the number of comparisons (and motivate your answer). Is this an in-place algorithm?

(b) Suppose that the number of steps of an algorithm $T(n)$ with an input of n , has the recurrence relation

$$T(n) = 4T(n/2) + n^2$$

What is the asymptotic complexity class of this algorithm?

2. (10 points)

- (a) Suppose we have a minheap with at least three elements. What is the index of the smallest element bigger than the minimum in this minheap? Explain your answer.
- (b) Given a non-empty binary search tree with unique elements. Give an algorithm (in words or in pseudocode) that yields the smallest element bigger than the minimum.

3. (10 points)

A number of tracks are to be burnt on a CD. Assume that a choice can be made from n tracks, with a track time of t_i minutes for track i . The CD is to be filled for as far as possible, i.e. with the maximum possible number of minutes of music. Assume that a CD contains at most 80 minutes of music (if it is completely filled).

- (a) Suppose that $C(i, k)$ is the minimum remaining time (i.e. the number of unused minutes) when k minutes are to be filled with tracks from the collection $\{1, \dots, i\}$. Explain that

$$C(i, k) = \min\{C(i-1, k), C(i-1, k - t_i)\}$$

where $C(i, k) = \infty$ for $k < 0$.

What is the value of $C(0, k)$ for $k \geq 0$?

- (b) Give a polynomial algorithm based on dynamic programming that calculates the maximum number of minutes that can be burnt on the CD.