Course	: Statistical Techniques for TCS/BIT	Date	: Thursday 21 December, 2023
Module	: 6	Time	: 08:45 – 11:00 (2 hours)
Course code	: 202001033	Reference	: Intelligent Interaction Design TCS/BIT (2023-1B)

A

Statistical Techniques for TCS/BIT

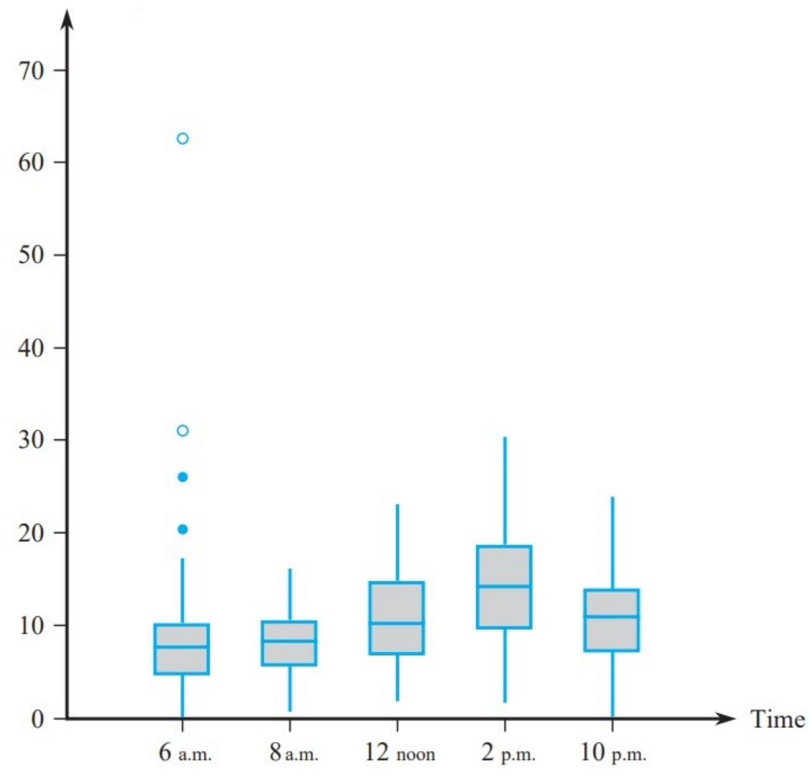
Exam

Instructions

This examination comprises 11 exercises and is divided into two sections: Final answer questions and Open questions. For the Final answer questions, record only your answers. For the Open questions, provide explanations or motivation for all your answers and present all relevant solutions. Provide your responses rounded to *three decimal places*, where applicable. Submit all your responses on the designated answer sheet.
Supplementary scratch paper is accessible for your convenience (this will not be assessed).
Ensure to hand in your answer sheet and to return the formula sheet and probability tables, which are provided separately.

Final answer questions

1. The developers of a smart home voice command app carried out a study to examine the app usage frequency in 20 smart houses during 30-minute time intervals at different times throughout the day. For example, the frequency recorded at 6 a.m. was assessed from 5:45 a.m. to 6:15 a.m. Below, you'll find the corresponding comparative box-plot depicting the frequency counts for five distinct times of the day. Answer the following statements **(A)** to **(F)** about the above comparative box-plot depicting the frequency counts with either true (T) or false (F). [2 pt]



(A) There are both extreme and mild outliers at 6 a.m. **TRUE**

(B) The mean app usage frequency at 12 noon is 10. **FALSE**

(C) Variability increases until 2 p.m. **TRUE**

(D) At 8 a.m., the app usage frequency is the most consistent. **TRUE**

(E) Distributions at 8 a.m. and 2 p.m. are not symmetric. **FALSE**

(F) The largest app usage frequency at 2 p.m. is greater than the largest app usage frequency at 10 p.m. **TRUE**

2. Let X_1, X_2, X_3, and X_4 be be a random sample of observations from a population with mean μ and variance σ^2. [2 pt]

Consider the following two point estimators of μ: **(C)**

$T_1 = 0.10X_1 + 0.40X_2 + 0.40X_3 + 0.10X_4$,

$T_2 = 0.20X_1 + 0.30X_2 + 0.30X_3 + 0.20X_4$.

I. Which of the following is true?

- **(A)** T_1 is a biased and T_2 is an unbiased estimator of μ.
- **(B)** T_1 is an unbiased and T_2 is a biased estimator of μ.
- **(C)** **Both T_1 and T_2 are unbiased estimators of μ.**
- **(D)** Both T_1 and T_2 are biased estimators of μ.
- **(E)** We cannot tell about the bias of T_1 and T_2 with the given information.

II. Which of the following is true? **(B)**

- **(A)** $Var(T_1) = Var(T_2)$
- **(B)** $\boldsymbol{Var(T_1) > Var(T_2)}$
- **(C)** $Var(T_1) < Var(T_2)$
- **(D)** We cannot tell the relationship between $Var(T_1)$ and $Var(T_2)$ with the given information.

III. Which of the following is true regarding the Mean Squared Error (MSE)?**(A)**

- **(A)** **T_2 is a better estimator of μ than T_1.**
- **(B)** T_1 is a better estimator of μ than T_2.
- **(C)** We cannot tell whether T_1 or T_2 is a better estimator of μ with the given information.

3. We are testing if the mean amount of liquid dispensed by a machine is less than 1000 units with $\alpha = 0.05$, the standard deviation is given $\sigma = 24$ units. We're assessing the risk of not detecting under-dispensing by 10 units using a sample of $n = 40$ bottles. The hypotheses of this research are: $H_0 : \mu = 1000$, $H_a : \mu < 1000$. We have the following Rejection Rule: "Reject H_0 if $Z < -1.645$" And $z = \frac{\bar{x}-1000}{\frac{24}{\sqrt{40}}} = -1.645$ gives $\bar{x} = 993.76$ [2 pt]

In this test, with a true mean of 990 units,which of the following is the correct computation of Type II error?**(C)**

- **(A)** $P(\bar{x} > 993.76 | \mu = 1000)$
- **(B)** $P(\bar{x} < 993.76 | \mu = 1000)$
- **(C)** $\boldsymbol{P(\bar{x} > 993.76 | \mu = 990)}$
- **(D)** $P(\bar{x} < 993.76 | \mu = 990)$
- **(E)** $P(\bar{x} > 990 | \mu = 993.76)$
- **(F)** $P(\bar{x} < 990 | \mu = 993.76)$
- **(G)** $P(\bar{x} > 990 | \mu = 1000)$
- **(H)** $P(\bar{x} < 990 | \mu = 1000)$

4. The lead researcher in a laboratory believes that the average time for participants to complete a cognitive task on the smart devices is 14 minutes. Assume that task completion time follows a normal distribution with a standard deviation of 3.4 minutes. The researcher monitors the completion times of 25 participants and finds that the average time for task completion was 11.6 minutes. [2 pt]

I. What are the appropriate null and alternative hypotheses? (**C**)

(A) $H_0 : \mu \geq 14$ and $H_1 : \mu < 14$.

(B) $H_0 : \mu \leq 14$ and $H_1 : \mu > 14$.

(C) $H_0 : \mu = 14$ and $H_1 : \mu \neq 14$.

(D) $H_0 : \mu \geq 14$ and $H_1 : \mu = 14$.

II. Which statistical test should the researcher consider to test their statistical hypothesis for the above mentioned claim? (**A**)

(A) One sample t-test for the μ when σ^2 is unknown

(B) One sample test for the μ when σ^2 is known

(C) Paired samples t-test for the μ

(D) Chi-square test for the σ^2

III. Which decision below should the researcher make for testing the above claim with a significance level of $\alpha = 5\%$? (**G**)

(A) $z_{critical} < z_{observed}$ therefore Fail to reject H_0

(B) $z_{critical} > z_{observed}$ therefore Fail to reject H_0

(C) $t_{critical} > t_{observed}$ therefore Fail to reject H_0

(D) $t_{critical} < t_{observed}$ therefore Fail to reject H_0

(E) $z_{critical} > z_{observed}$ therefore Reject H_0

(F) $z_{critical} < z_{observed}$ therefore Reject H_0

(G) $\boldsymbol{t_{critical} > t_{observed}}$ **therefore Reject** $\boldsymbol{H_0}$

(H) $t_{critical} < t_{observed}$ therefore Reject H_0

5. A study on 22 engineering students categorized them into two groups: Bike Commuters (12 students, mean distance 28.5 km, variance 18.1 km^2) and Public Transit Commuters (10 students, mean distance 32.2 km, variance 20.4 km^2). [2 pt]

I. Researchers plan an eight-step hypothesis test to assess if there's a significant difference in mean distances between the two groups. (**C**)

(A) $\frac{\bar{X}_{\text{Bike}} - \bar{Y}_{\text{Public}}}{\sqrt{S^2\left(\frac{1}{12}+\frac{1}{10}\right)}}$ with $S^2 = \frac{12}{22}S_X^2 + \frac{10}{22}S_Y^2$

(B) $\frac{\bar{X}_{\text{Bike}} - \bar{Y}_{\text{Public}}}{\sqrt{S^2\left(\frac{1}{12}+\frac{1}{10}\right)}}$ with $S^2 = \frac{11}{21}S_X^2 + \frac{9}{21}S_Y^2$

(C) $\frac{\bar{X}_{\text{Bike}} - \bar{Y}_{\text{Public}}}{\sqrt{S^2\left(\frac{1}{12}+\frac{1}{10}\right)}}$ with $S^2 = \frac{11}{20}S_X^2 + \frac{9}{20}S_Y^2$

(D) $\frac{\bar{X}}{\frac{S}{\sqrt{20}}}$

(E) $\frac{Z}{\frac{S_Z}{\sqrt{22}}}$

(F) $\frac{Z}{\frac{S_Z}{\sqrt{21}}}$

(G) $\frac{Z}{\frac{S_Z}{\sqrt{20}}}$

II. Which of the following represents the distribution for the scenario described above? (**A**)

(**A**) t_{20} Under H_0 (**B**) t_{21} Under H_0 (**C**) t_9 Under H_0 (**D**) t_{22} Under H_0 (**E**) t_{11} Under H_0

6. Suppose in a local city, 53 percent of the population favors the implementation of a new tram line. A simple random sample of 300 residents is surveyed. We define X as the number of residents favoring the new tram line. $X \sim B(n, p)$ where $n = 300$ and $p = 0.53$. Which of the following is true? (**B**) [1 pt]

(**A**) Since $np > 5$ and $n(1-p) > 5$, by Central Limit Theorem the binomial distribution $Bin(159, 8.644^2)$ can be used to describe the distribution of X.

(**B**) **Since $np > 5$ and $n(1-p) > 5$, by Central Limit Theorem the normal approximation $N(159, 8.644^2)$ can be used to describe the distribution of X.**

(**C**) Since $np > 5$ and $n(1-p) > 5$, by Central Limit Theorem the normal approximation $N(300, 0.53)$ can be used to describe the distribution of X.

(**D**) Since $np > 5$ and $n(1-p) > 5$, X $\sim N(159, 8.644^2)$.

(**E**) Since $np > 5$ and $n(1-p) > 5$, X $\sim Bin(159, 8.644^2)$.

7. In attempting to assess the effectiveness of solar panels installed on rooftops using technology A as opposed to technology B, and aiming to form a comprehensive judgment on the relative efficiency of these technologies, the researcher discovers a significant violation of the normality assumption. In this scenario, which of the following statistical methods could the researcher potentially employ?(**D**) [1 pt]

(**A**) Indepentent samples t-test

(**B**) Paired samples t-test

(**C**) Chi Square test

(**D**) **Wilcoxon rank sum test**

(**E**) Sign test on the Median of the differences

(**F**) Shaphiro Wilk's test

Open questions

8. The presented data table below outlines the average counts of successful gestures performed by each participant during a hands-free interaction session within a virtual reality (VR) environment. Each participant underwent testing in four measurements, and the average value is provided in the table below. This research was conducted at the Center for Human-Computer Interaction at the University of Twente and involved a total of 43 participants. The gesture success counts are organized from the least to the most successful interactions. The classical numerical summaries and graphical presentations, are provided below.

2.0	2.2	2.9	3.0	3.7	3.9	4.0	4.1	4.4	4.8
5.8	5.9	6.3	6.8	6.9	7.2	7.4	7.5	7.6	8.0
8.4	9.0	9.2	9.2	9.4	9.8	9.9	10.0	10.1	10.4
10.5	11.1	11.2	11.3	11.3	11.5	12.0	13.0	15.4	15.4
15.8	19.6	22.7							

(a) Average Gesture Success Counts

Statistical Measure	Value
Sample size	43
Sample mean	8.851
Sample standard deviation	4.484
Sample variance	20.109
Sample skewness	0.893
Sample kurtosis	4.298

(b) Statistical Summary

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Invullen in blokletters/To be completed by student

Cursusnaam/*Coursename*		Datum/*Date*	Bladnr./*Page no.*
Cursuscode/*Coursecode*			
Studentnr./*Student no.*	Voorl./*Initials*	Opleiding/*Programme*	Groepnr./*Group no.*
Naam/*Name*			

Q8)

I) Five number summary is as follows:

minimum : 2.0

Q1 : 25th quartile : $\frac{25}{100} * 43 = 10.75 \Rightarrow Q_1 = X_{11} = 5.8$

median : 50th quartile : $\frac{50}{100} * 43 = 21.5 \Rightarrow median = X_{22} = 9.0$

Q3 : 75th quartile : $\frac{75}{100} * 43 = 32.25 \Rightarrow Q_3 = X_{33} = 11.2$

maximum : 22.7

Q8)

II) $1.5 IQR \Rightarrow 1.5 * (Q_3 - Q_1) = 1.5 * (11.2 - 5.8)$
$= 1.5 * 5.4 = 8.1$

1.5 IQR : $(Q_1 - 8.1, Q_3 + 8.1) = (5.8 - 8.1, 11.2 + 8.1)$

$= (-2.3, 19.3)$ is the 1.5 IQR.

Accordingly we have two outliers that are 19.6 and 22.7.

Q8) III)

a) The skewness coefficient (0.893) is greater than the reference value which is 0 for the normal distribution.

This indicates skewness to the right.

The kurtosis coefficient (4.298) is also greater than the reference value which is 3 for the normal distribution. Whereas kurtosis coefficient is significantly smaller than the reference value (9) of the exponential distribution.

b) The histogram is not perfectly symmetric nor bell-shaped. It's skewed to the right, while the distribution does not look strongly non-normal and there are several peaks.

c) The normal QQ-plot shows no major deviation from the line $y=x$, except for the larger observations (which were found to be outliers in part II).

Overall, there's some doubt about the normal distribution model.

Q8) IV) According to the Shaphiro Wilk's table with

$n=43$ and $\alpha=10\%$ we reject the null hypothesis

of a normal distribution if $W \leq c = 0.951$.

Since $W=0.944$, $W<c$ and is in the rejection region. We reject the null hypothesis (H_0).

Normal distribution does not apply to the "average counts of successful gestures" at a 10% significance level.

Q9)

I) The assumptions are that the two samples are independent and normally distributed.

II) If the normality assumption is violated, then we cannot apply the parametric tests. In such a case, we would apply the non-parametric alternative for independent samples t-test which is Wilcoxon's rank test.

III)

1) $X_1, \dots, X_8$: response time of system X
$Y_1, \dots, Y_5$: response time of System Y

$X_i \sim N(\mu_x, \sigma^2)$ and $Y_i \sim N(\mu_y, \sigma^2)$

2) $H_0: \mu_x - \mu_y = 0$ $H_1: \mu_x - \mu_y > 0$ $\alpha = 0{,}05$

3) Test statistic : $T = \frac{(\bar{X} - \bar{Y})}{\sqrt{S^2(\frac{1}{8} + \frac{1}{5})}}$ and $S = \frac{7S_x^2 + 4S_y^2}{8+5-2}$

4) T has t_{11} distribution under Ho

5) Observed: $s^2 = \frac{7}{11} * (25.11)^2 + \frac{4}{11} * (17.84)^2 \approx 516{,}97$

and $t = \frac{110.5 - 100.4}{\sqrt{516.97 * (\frac{1}{8} + \frac{1}{5})}} \approx 0{,}779$

6) This is a right-sided test and we reject Ho if $T \geq c$ with $c = 1.796$ from the t_{11}-table.

7) $t = 0{,}779$ is not in the critical area, we reject Ho.

8) With a significance level of 5% the expected response time of System X is not greater than that of System Y.

10) We will apply (Pearson's) χ^2 [Chi-squared] test. on a distribution with 4 categories.

1) The numbers N_1, N_2, N_3, N_4 are multinomially distributed with total number $n = 200$ and corresponding probabilities with p_1, p_2, p_3, p_4.

2) We test $H_0: p_1 = p_2 = p_3 = p_4 = 0{,}25$
against $H_1: p_i \neq 0.25$ for at least one of the $i = 1,2,3,4$
with $\alpha = 0{,}05$

3) Test statistic: $\chi^2 = \sum_{i=1}^{4} \frac{(N_i - E_0 N_i)^2}{E_0 N_i}$ with

$E_0 N_i = 200 * 0{,}25 = 50$; $i = 1,2,3,4$

4) Under H_0, χ^2 has a Chi-squared distribution

with $df = k - 1 = 3$

5) Observed value:

$$\chi^2 = \frac{(55-50)^2}{50} + \frac{(40-50)^2}{50} + \frac{(59-50)^2}{50} + \frac{(46-50)^2}{50}$$

$= 4.44$

6) We reject H_0 if $\chi^2 \geq c$ where $c = 7.81$ from the χ^2_3 - table (with $\alpha = 0{,}05$).

7) The observed value 4.44 is not in the critical area, we do not reject H_0.

8) We do not have sufficient evidence, at a 5% significance level, that the distribution accross the four preferences of ornament decorations shows equality.

Q11)

I) the dependent variable is:

Y: Financial effects of balancing work and childcare responsibilities

the independent variable is:

X: Age of each mother

II) $Y = \beta_0 + \beta_1 X + \varepsilon \qquad \varepsilon \sim N(0, \sigma^2)$

III) $H_0: \rho = 0$
$H_1: \rho \neq 0$
} with $\alpha = 5\%$ $\left\{ \rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \right\}$

IV) The prediction is $Y = \beta_0 + \beta_1 X$ with

$H_0: \beta_1 = 0$ versus $H_1: \beta_1 \neq 0$ $(\alpha = 5\%)$

since we have $p = 0.046 < \alpha = 0.05$ we reject the null hypothesis that the population slope is zero.

The conclusion is that: there's sufficient evidence to conclude that there's a significant relationship between the 'age of mothers' and the financial effects of balancing work and childcare responsibilities.