UNIVERSITY OF TWENTE.

Course : Calculus 1A
Module : 1A

Date : Friday 27 October, 2023
Time : 13:45 – 15:45 (2 hours)

B

Calculus 1A

Exam

Instructions

The answer form Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Final answer questions On the answer form, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer questions You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

⋆ ⋆ ⋆ The use of electronic devices is not allowed. ⋆ ⋆ ⋆

Questions

1. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

Define $P(-1,1,2)$, $Q(1,3,1)$, and $R(2,1,-1)$. Let $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$.

(a) Determine vector $\mathbf{w}$ which is in the direction of $\mathbf{u}$ and has the magnitude $|\mathbf{w}| = 2$. [1 pt]

(b) The point P lies in the plane W. It is also given that the plane W is parallel to the plane V given by $x + 4y - z = 10$. [1 pt]
Determine an equation for the plane W of the form $ax + by + cz = d$.

(c) Calculate the angle between $\mathbf{u}$ and $\mathbf{v}$. [2 pt]

(d) Calculate the vector projection of $\mathbf{u}$ onto $\mathbf{v}$. [2 pt]

2. The function $f(x)$ is defined as [2 pt]

$$f(x) = \begin{cases} \dfrac{\sqrt{x}-2}{x-4} & \text{for } x > 4 \\ a & \text{for } x \leq 4 \end{cases}$$

where $x \in \mathbb{R}$ and $a \in \mathbb{R}$. For which value of a is $f(x)$ continuous at $x = 4$?
Explain all your reasoning.

3. Define the function

$$h(x) = \frac{1}{x} + \ln x.$$

(a) What is the domain of h? Provide only your final answer. [1 pt]

(b) Find all critical points of the above function h. [2 pt]

(c) Find the absolute extreme values of h on the interval $[1/2, 4]$, and specify where the extreme values are attained (that is: specify all x_0 for which $h(x_0)$ is an absolute maximum or minimum value Hint: the exact value of $\ln 1/2$ and $\ln 4$ is not needed, you may use $-1/2 < \ln 1/2 < 0$ and $2 < \ln 4$. [2 pt]

4. Evaluate the following limit: [3 pt]

$$\lim_{x \to 0^+} \frac{\ln(1 - e^x)}{2\ln(x)}.$$

5. The following piecewise function is given:

$$f(x,y) = \begin{cases} \dfrac{3xy}{x^2 + 2y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0). \end{cases}$$

(a) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Explain your reasoning. [2 pt]

(b) Is $f(x,y)$ continuous at $(0,0)$? Explain your reasoning. [1 pt]

6. Find an equation for the tangent plane to the graph of the function [3 pt]

$$f(x,y) = \sqrt{y - x^2}$$

at the point $(1, 2, f(1,2))$. Simplify the equation as much as possible.

Total: 22 pt