

Do not fill in!

Exercise	1	2	3	4	5	6	Σ
Points							

Name:

Student number:

Course : **Discrete Mathematics for Computer Science; Exam**

Date : November 4th, 2025

Time : 8:45 – 10:45

Motivate all your answers and give proofs if applicable, unless otherwise stated.

Write your solutions in the designated boxes.

The use of electronic devices is not allowed.

1. [3+2+1 pt]

(a) Let, the following symbols represent these primitive statements

Symbol	Statement
p	I have to work
q	it rains
r	I go to the amusement park
s	I go to the football match
t	I go to the museum

Determine for each of the statements in symbolic form if it is equivalent to the following natural language statement.

If I don't have to work and it doesn't rain, I will go to the amusement park or to the football match. If I don't have to work and it does rain, I will go to the museum.

Statement	Equivalent
$[(\neg p \wedge \neg q) \rightarrow (r \vee s)] \wedge [(\neg p \wedge q) \rightarrow t]$	yes / no
$[(\neg p \wedge \neg q) \rightarrow (r \vee s)] \vee [(\neg p \wedge q) \rightarrow t]$	yes / no
$\neg p \rightarrow [(\neg q \rightarrow (r \vee s)) \wedge (q \rightarrow t)]$	yes / no
$\neg p \rightarrow [(\neg q \rightarrow (r \vee s)) \vee (q \rightarrow t)]$	yes / no
$\neg p \rightarrow [r \vee s \vee t]$	yes / no

You do not need to motivate your answer.

*Grading :
-1.5 pts per
wrong/missing
answer.
No less than
0 points.*

(b) Prove the following equivalence or give a counterexample.

$$(p \wedge q) \vee \neg(\neg p \vee q) \iff p$$

For a proof, use the **laws of logic**. Use only one law for each step.

(c) Prove the following equivalence or give a counterexample.

$$(p \wedge q) \vee (\neg r \wedge s) \iff (\neg r \vee q) \wedge (p \vee s)$$

For a proof, use the **laws of logic**. Use only one law for each step.

Grading: $\frac{1}{2}$ pt for a correct counter-example.
$\frac{1}{2}$ pt for verifying it.

2. [3+3 pt]

(a) Prove the following statement

$$[(p \vee q) \wedge (\neg p \vee q) \wedge (q \rightarrow r) \wedge \neg(q \wedge p) \wedge \neg s] \rightarrow [(p \wedge r) \rightarrow s]$$

Use the **rules of inference** and/or the **laws of logic**. Use only one rule or law for each step and explain which previous steps you used.

Step	Steps used
Grading comment: Intermediate steps do <u>not</u> give partial points.	
Invented premises, clear short cuts (i.e., writing the conclusion after getting stuck), and inference of incorrect statements to short cut to the conclusion result in 0 points.	

(b) Verify the validity of the following argument.

$$\frac{\begin{array}{l} \forall x [r(x) \vee s(x)] \\ \exists x [r(x) \leftrightarrow s(x)] \\ \forall x [r(x) \rightarrow p(x)] \\ \forall x [s(x) \rightarrow q(x)] \end{array}}{\therefore \exists x [p(x) \wedge q(x)]}$$

For verification use the **rules of inference** and/or the **laws of logic**. Use only one rule or law for each step and explain which previous steps you used.

[illegible]

3. [3+3 pt]

(a) For each $i \in \mathbb{Z}^+$, let $A_i = \{x \in \mathbb{R} \mid (x^2 < i) \wedge (x > -1 - \frac{1}{i})\}$.

Determine $\bigcap_{i=3}^{\infty} A_i$.

(b) Consider a universe $\mathcal{U}$ and let $A, B, C \subseteq \mathcal{U}$. Either prove using *proof by contradiction*, or disprove the following claim:

$$\text{If } A - (B \cup \overline{C}) \subseteq B, \text{ then } A \cap C \subseteq B.$$

A direct proof
gives (at most)
1 pt.

↑ important - it does not hold for all $x \in A \cap C$.

4. [3+3 pt]

(a) Identify the smallest possible $n_0 \in \mathbb{Z}^+$, such that for all $n \in \mathbb{Z}^+$ with $n \geq n_0$ the following holds:

$$2^n \geq n^2.$$

Prove that n_0 is indeed the smallest integer such that the above holds.

Comment: An induction step from $k=2$ to $k=3$ cannot work (since $2^3 < 3^2$).

5. [2+2+2 pt] Recall that for a function $h : A \rightarrow B$, A is its domain and B is its co-domain.

- (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Prove that f is not one-to-one. Redefine its domain so that it becomes one-to-one, and prove that this is the case.

- (b) Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = e^x$. Prove that g is not onto. Redefine its co-domain so that it becomes onto, and prove that this is the case.

- [illegible]

6. [3+3 pt]

- (a) Let R_1 be a relation on A that is symmetric and transitive and furthermore satisfies the property that for every $a \in A$ there exists a $b \in A$ such that $(a, b) \in R$. Is R_1 an equivalence relation? Prove it or provide a counterexample.

- (b) Consider the relation R_2 on $\mathbb{Z}^+$, defined by

$$R_2 = \{(a, b) \mid a \text{ and } b \text{ have the same number of digits (in decimal)}\}.$$

Show that R_2 is an equivalence relation and give all corresponding equivalence classes. We count the digits of integers without leading zeroes.

Total: 36 points

