

Course : **Discrete Mathematics for Computer Science; Exam**
 Date : Tuesday, October 29th, 2024
 Time : 8:45–10:45

Motivate all your answers and give proofs if applicable, unless otherwise stated.
Write your solutions in the designated boxes.
The use of electronic devices is not allowed.

1. [1+2+3pt] *You do not need to motivate your answer for parts (a) and (b) of this question.*

(a) Let the universe comprise all students at the UT. We define the following open statements

- * $d(x)$ student x is enrolled in the course Discrete Mathematics
- * $s(x)$ student x studied the course material and did the exercises for Discrete Mathematics.
- * $e(x)$ student x passes the exam in Discrete Mathematics

Express the following statement in symbolic form:

All enrolled students of the course Discrete Mathematics pass the exam if they studied the course material and did the exercises.

Do not fill in!

Exercise	1	2	3	4	5	6	Σ
Points							

- * $b(x)$ guest x came to the party by bike
- * $c(x)$ guest x came to the party by car

All guests to the party came by bike except one, who came by car.

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2. [3+3pt]

(a). Verify the validity of the following argument using laws of logic and rules of inference. Use only one rule or law for each step and explain which previous steps you used.

$$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$$

[illegible]

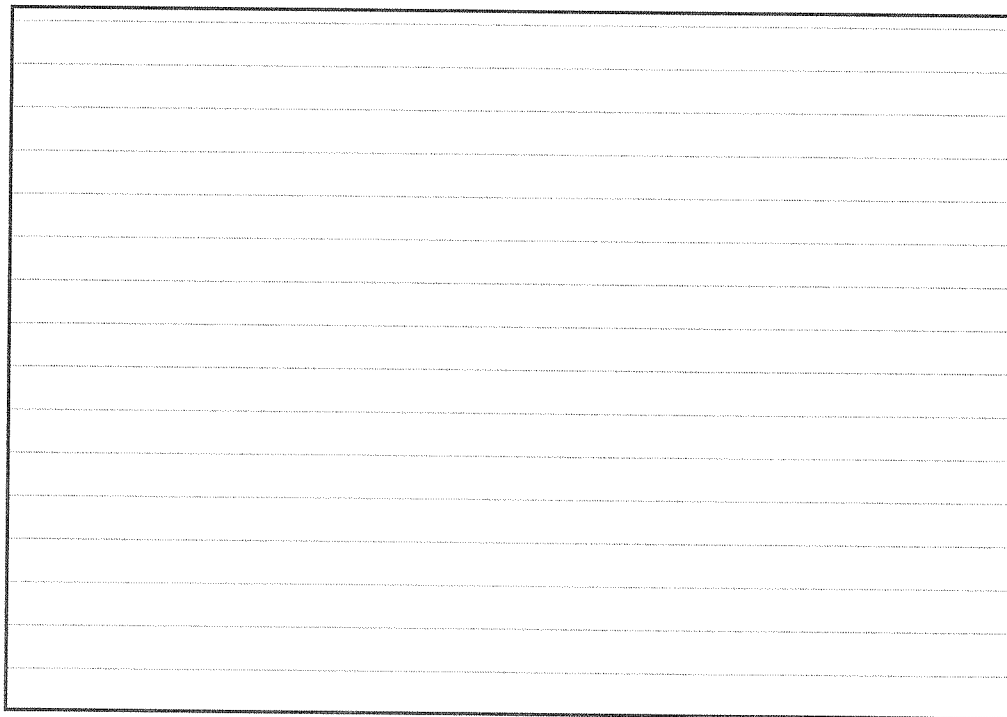
- Use only one rule or law for each step and explain which previous steps you used. Give an explanation if you introduce or use new variables or quantifiers.

[illegible]

3. [2+4pt]

(a) Let $A, B, C \subseteq \mathcal{U}$. Prove or disprove the following:

$$A - C = B - C \Rightarrow A = B$$

A large rectangular box with horizontal lines, intended for a proof or disproof.

(b) Let $A, B \subseteq \mathcal{U}$. Prove or disprove the following:

$$A \subseteq B \Leftrightarrow A \cap B = A$$

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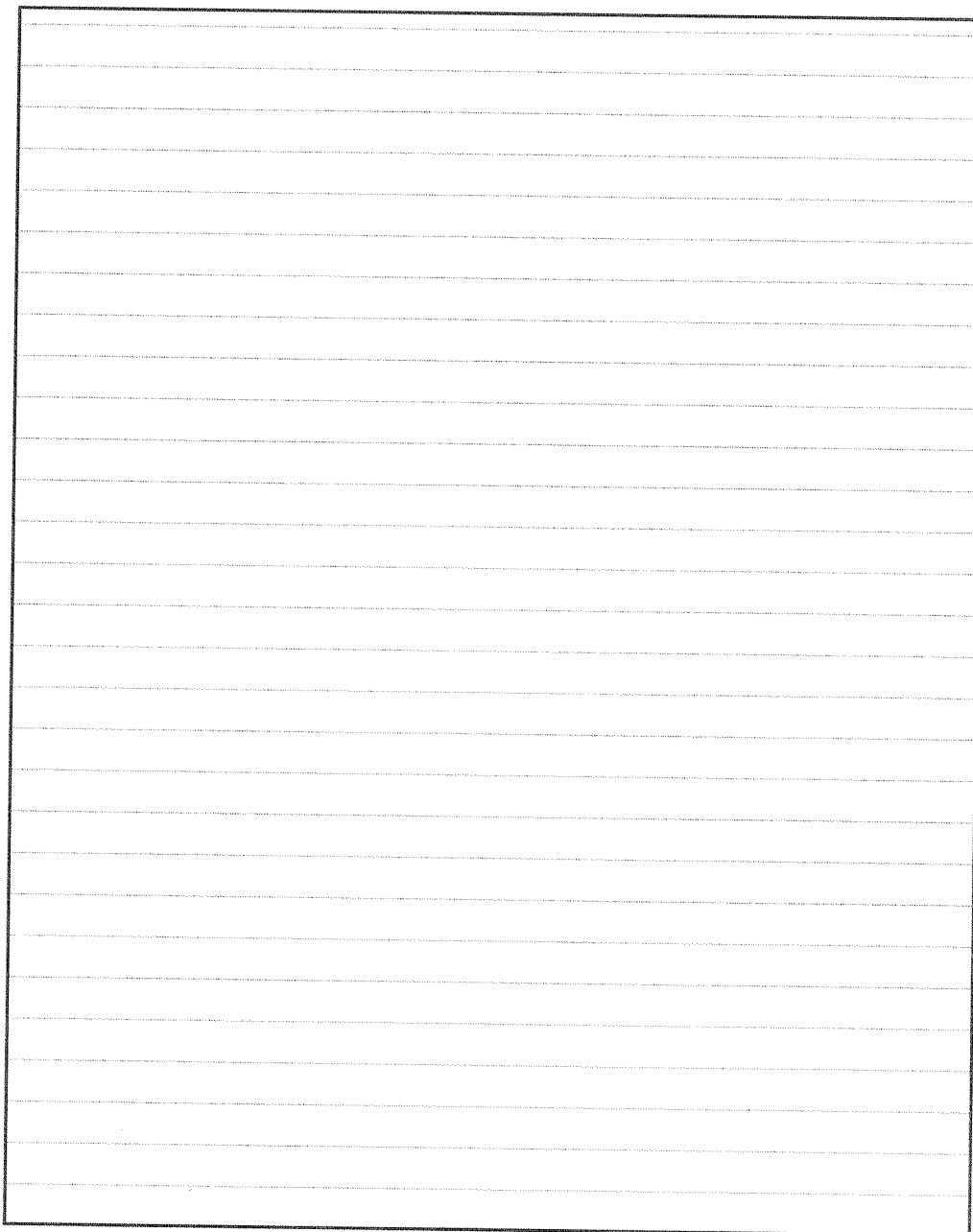
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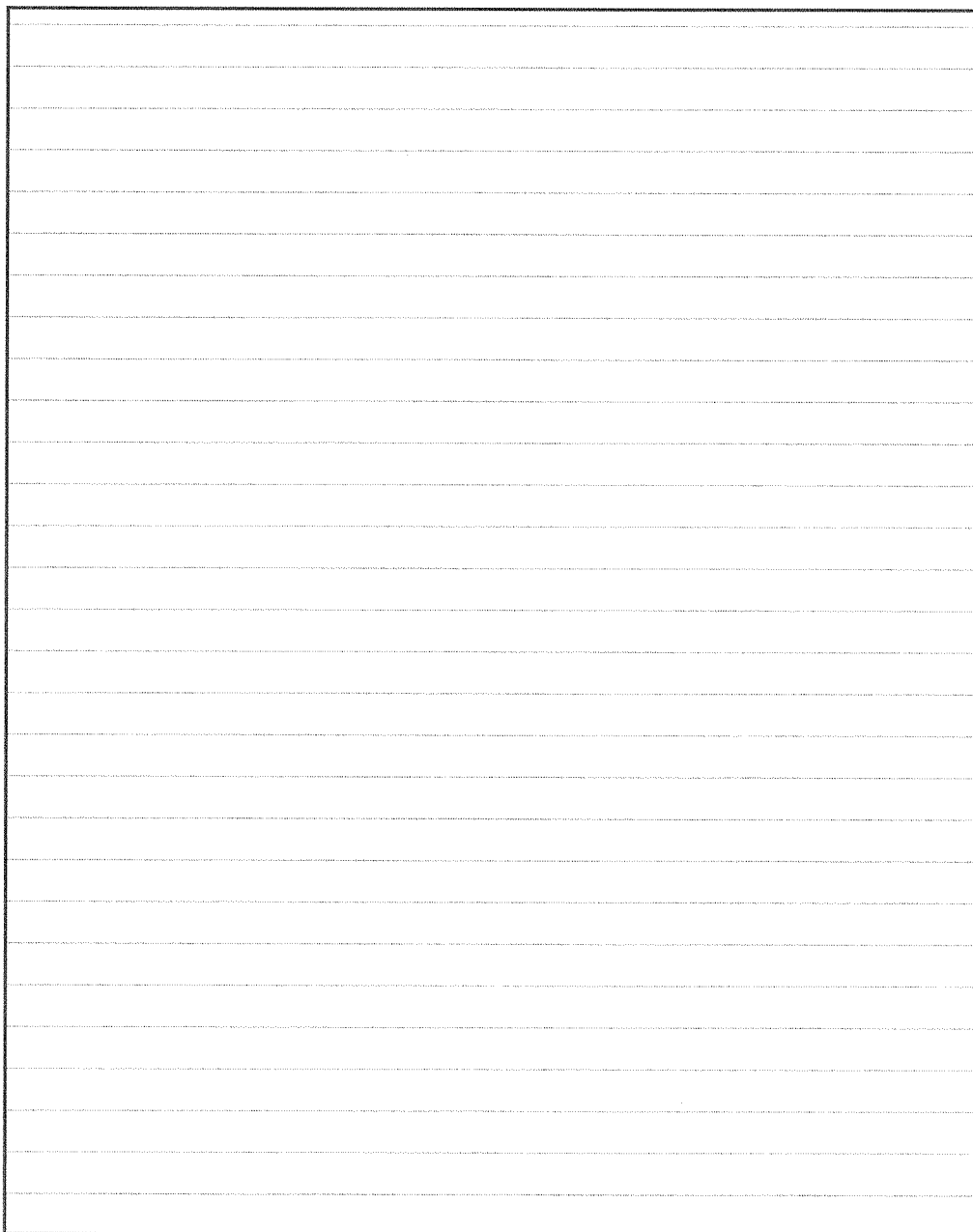
4. [3+3pt]

(a) For all $n \in \mathbb{Z}^+$ with $n \geq 12$, show that n can be written as a sum of 3's and/or 7's.



(b) Prove that for any integer $n \geq 2$,

$$1 + 4 + 7 + 10 + 13 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}.$$



5. [2+2+1+1pt]

Let $f : \mathbb{Z} \rightarrow \mathbb{Z}^+$ be defined by

$$f(x) = \begin{cases} 2x + 1, & \text{if } x \geq 0 \\ -2x & \text{otherwise.} \end{cases}$$

(a) Prove or disprove: f is one-to-one.

(b) Prove or disprove: f is onto.

(c) Prove or disprove: f is invertible.

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There is no handwriting or other markings on the paper.

6. [2+2+2pt]

(a) Let $R_1 \subseteq A \times A$, with $A = \{-2, 1, 2, 3, 5\}$ be given by

$$R_1 = \{(-2, -2), (-2, 1), (-2, 2), (1, -2), (1, 1), (1, 2), (1, 5), (2, -2), \\ (2, 1), (2, 2), (2, 5), (3, 5), (5, 1), (5, 2), (5, 3), (5, 5)\}.$$

Is R_1 transitive, symmetric, antisymmetric, and/or reflexive? Justify your answers.

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(b) Let W be a set with $|W| \geq 2$ and let R_2 be the binary relation on $\mathcal{P}(W)$ defined by:

$$(A, B) \in R_2 \Leftrightarrow A \cap B \neq \emptyset.$$

Is R_2 transitive, symmetric, antisymmetric, and/or reflexive? Justify your answers.

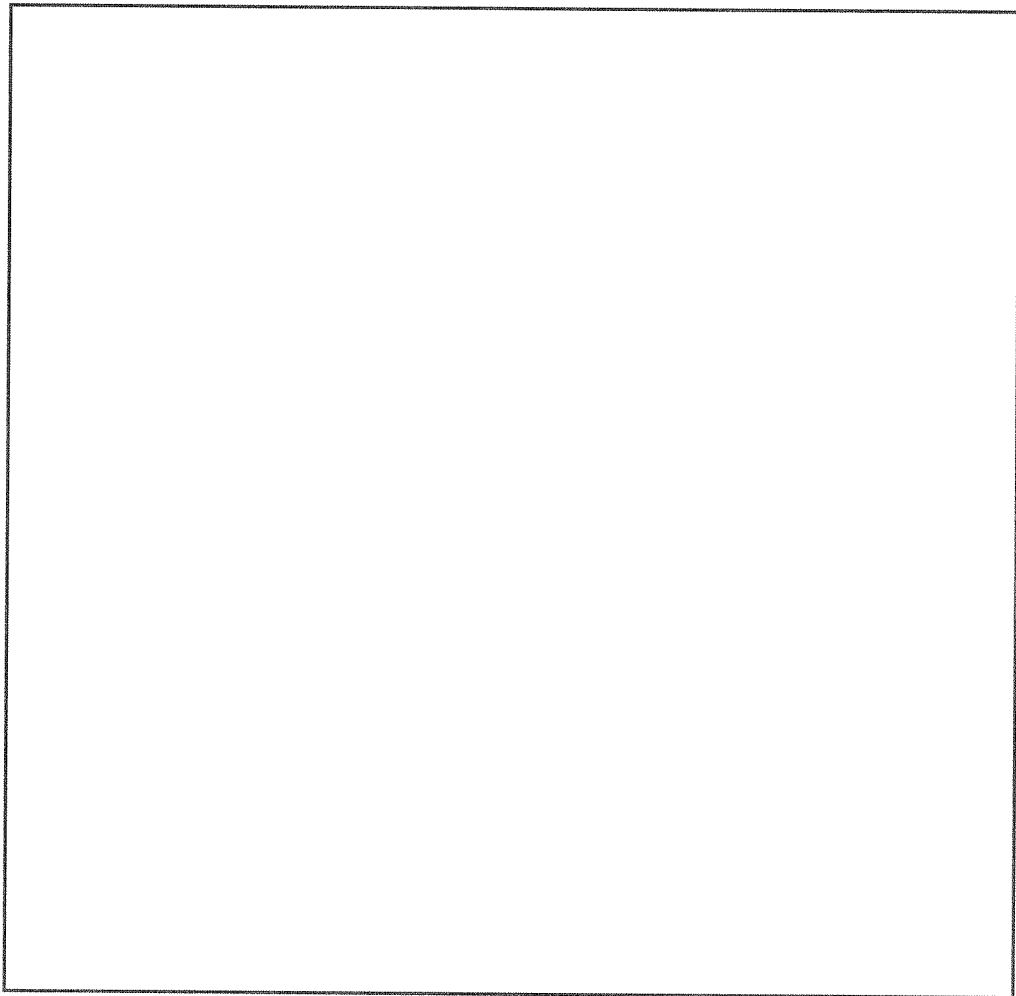
This image shows a single sheet of white paper with horizontal blue or grey ruling lines. A vertical margin line is present on the left side, creating a narrow left margin. The paper appears to be from a notebook or a set of legal pads. There are no markings, text, or drawings on the page.

(c) Let R_3 be the (partial order) relation on \mathbb{Z}^+ defined by

$$R_3 = \{(a, b) \mid a \text{ divides } b\}.$$

Draw the Hasse diagram representing R_3 , on the set

$$A = \{1, 2, 3, 4, 6, 8, 12, 24\}.$$



A large rectangular box with a black border, containing approximately 25 horizontal lines for writing. The lines are evenly spaced and extend across the width of the box.

Total: 36 points