UNIVERSITY OF TWENTE.

Course	: Calculus 1B	Date	: February 24, 2025
Module	: 1B	Time	: 18:15 – 20:15
Course code	: 20200119[4–9], 20201201	Reference	: TEST2

A

Calculus 1B

Exam

Instructions

This exam contains 10 questions. You shall use the attached *answer form* to submit your answers.

- For questions 1–5, you are only required to fill in the **final answer** on the answer form.
- For questions 6–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. Any text outside the answer form will not be considered.
If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.
Do not write with red pen or pencil. Do not use correction fluid or tape.
The use of electronic devices is not allowed!

Final answer questions

Write only your final answer on the answer form.

1. Let R be the area of the region enclosed by the line $y = x + 2$ from above, by the graph of the function $f(x) = \sqrt{4 - x^2}$ from below, and by the line $x = 2$ on the right. [2 pt]

On your answer form, select **all** expressions that equal R.

(A) $\displaystyle\int_0^2 \left(x + 2 - \sqrt{4 - x^2}\right) dx$

(B) $\displaystyle\int_0^2 \left(x + 2 + \sqrt{4 - x^2}\right) dx$

(C) $\displaystyle\int_0^1 \left(x + 2 - \sqrt{4 - x^2}\right) dx + \int_1^2 \left(x + 2 - \sqrt{4 - x^2}\right) dx$

(D) $\displaystyle\int_{-1}^2 \left(x + 2 - \sqrt{4 - x^2}\right) dx - \int_{-1}^0 \left(x + 2 - \sqrt{4 - x^2}\right) dx$

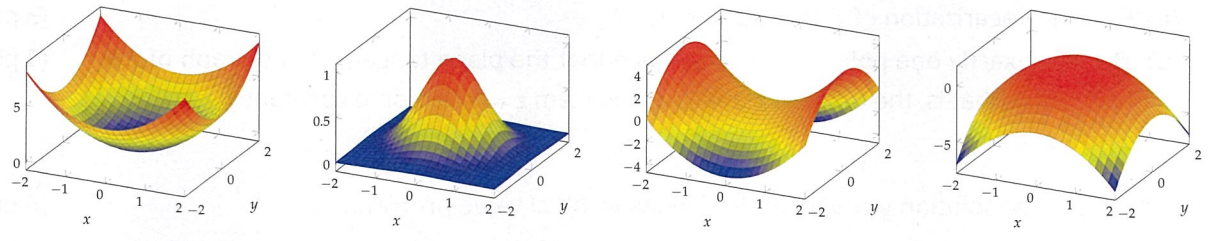
2. Find the derivative $f'(x)$ of the function f defined by [2 pt]

$$f(x) = \int_{x^4}^3 \cos(t) \arctan(t)\, dt\,.$$

3. In Figure 1 below, the following functions have been graphed: [2 pt]

$$f(x,y) = e^{-x^2-y^2}\,, \quad g(x,y) = x^2 - y^2\,, \quad h(x,y) = 1 - x^2 - y^2\,, \quad k(x,y) = x^2 + y^2\,.$$

For each function, match it with the figure (A, B, C, or D) that shows its graph.



(A) (B) (C) (D)

Figure 1: The four graphs of Question 3.

4. Find the four complex numbers z that satisfy the equation [2 pt]

$$z^4 - 16i = 0\,.$$

Give your solutions in the form $r\,e^{i\theta}$ with $r, \theta \in \mathbb{R}$.

5. The following equation in polar coordinates describes a circle: [2 pt]

$$r = 8\cos\theta\,.$$

Find the circle's radius and the Cartesian coordinates of its center.

Open questions

Provide a full calculation and argumentation on the answer form.

6. Evaluate the following integral: [5 pt]

$$\int_{-\infty}^{0} e^{2x}\sqrt{1+e^x}\,dx\,.$$

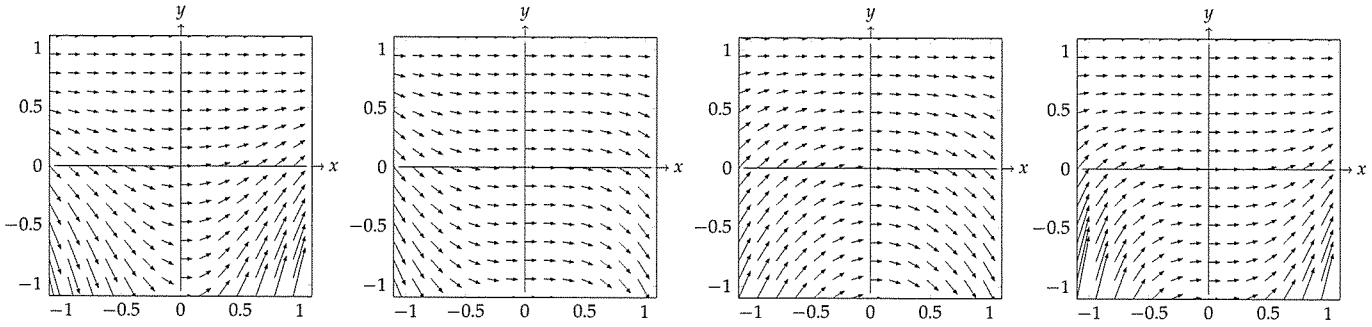
7. Evaluate the following integral: [4 pt]

$$\int (x^2+\pi)\cos(x+\pi)\,dx\,.$$

8. **(a)** Exactly one of the four slope fields in Figure 2 below is a slope field for the first-order differential equation $y' = x(y-1)^2$. Which one? Clearly motivate your answer. [2 pt]

(b) Find the unique solution $y = y(x)$ to the following initial value problem: [4 pt]

$$y' = x(y-1)^2\,, \qquad y(0) = \tfrac{1}{2}\,.$$



(A) (B) (C) (D)

Figure 2: The four slope fields of Question 8(a).

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$f(x,y) = xy^2 + xe^x\,.$$

(a) Find the linearization of f at the point $(1,-2)$. [3 pt]

(b) There is exactly one point $(a,b,f(a,b))$ such that the plane tangent to the graph of f is horizontal (that is, the tangent plane is of the form $z = c$ for some constant $c \in \mathbb{R}$). Find a and b. [2 pt]

10. Find the unique solution $y = y(x)$ to the following initial value problem: [6 pt]

$$4y'' + 4y' + 5y = 26e^x\,, \qquad y(0) = 0\,, \qquad y'(0) = 6\,.$$

Total: 36 pt