UNIVERSITY OF TWENTE.

Course	: Calculus 1B	Date	: January 31, 2025
Module	: 1B	Time	: 08:45 – 10:45
Course code	: 202001200	Reference	: IEM-RESIT

A

Calculus 1B

Exam

Instructions

This exam contains 10 questions. You shall use the attached *answer form* to submit your answers.

- For questions 1–5, you are only required to fill in the **final answer** on the answer form.
- For questions 6–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. Any text outside the answer form will not be considered.
If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.
Do not write with red pen or pencil. Do not use correction fluid or tape.
The use of electronic devices is not allowed!

Final answer questions

Write only your final answer on the answer form.

1. Let a and b be real numbers with $a < b$, and let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function with

$$f(x) < 0 \text{ and } f'(x) > 0 \text{ for all } x \in [a, b]\,.$$

Questions 1(a) and 1(b) below are both about this function f.

(a) Let R be the **area** of the region bounded on the left by the line $x = a$, on the right by the line $x = b$, above by the x-axis, and below by the graph of f. On your answer form, indicate which of the following integrals equals R. (Select all the correct answers.) [1 pt]

$R =$

(A) $\int_a^b f(x)\,dx$

(B) $-\int_a^b f(x)\,dx$

(C) $\int_a^b -f(x)\,dx$

(D) $-\int_a^b -f(x)\,dx$

(E) $\int_b^a f(x)\,dx$

(F) $-\int_b^a f(x)\,dx$

(G) $\int_b^a -f(x)\,dx$

(H) $-\int_b^a -f(x)\,dx$

(b) For any positive integer n, let S_n denote the Riemann sum for f on the interval $[a, b]$ that is obtained by dividing $[a, b]$ into n equal subintervals and using the *left-hand endpoint* of each subinterval to evaluate f. For each of the following statements, determine whether it is true or false. [2 pt]

(A) $S_n < 0$ for every positive integer n

(B) $S_n < \int_a^b f(x)\,dx$ for every positive integer n

(C) $S_{100} < S_{10}$

2. Determine [1 pt]

$$\frac{d}{dx}\int_x^0 \frac{\sin\theta\, e^{\cos\theta}}{e^\theta + 7}\,d\theta\,.$$

3. Determine the second-order partial derivative $\dfrac{\partial^2 f}{\partial y^2}$ of the function f given by [2 pt]

$$f(x,y) = x\ln(xy^2+3) + e^{2y}\,, \qquad x > 0\,.$$

4. The following equation in polar coordinates describes a circle: [2 pt]

$$r = -10\sin\theta.$$

Find the circle's radius and the Cartesian coordinates of its center.

5. For each of the following equations, find all complex numbers z that satisfy the equation. Give your answers in Cartesian form (that is, in the form $x+yi$ with $x, y \in \mathbb{R}$).

(a) $z^4 = -16$. [2 pt]

(b) $z = 2\overline{z} + 5$. [1 pt]

(c) $z^2 - 7 = 0$. [1 pt]

Open questions

Provide a full calculation and argumentation on the answer form.

6. Evaluate the following integral: [4 pt]

$$\int \frac{x}{1+4x^4}\,dx\,.$$

7. Evaluate the following integral (pay attention to the domain of the integrand!): [5 pt]

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx\,.$$

8. Let f be the function given by

$$f(x,y) = \frac{18xy}{x^2+y^4+1}.$$

(a) Find an equation for the plane tangent to the graph of f at the point $(-2,1,f(-2,1))$. [3 pt]

(b) The linearization of f at the point $(1,0)$ is given by the function [1 pt]

$$L(x,y) = cy$$

for some constant c. Find c.

9. **(a)** Find the unique solution $y = y(x)$ to the following initial value problem: [4 pt]

$$\begin{cases} xy' + y^2 = 0\,, & x > 0\,, \\ y(e) = 2\,. \end{cases}$$

(b) Among all the solutions of the differential equation $xy' + y^2 = 0$, there is exactly one **constant** solution $y(x) = A$. Find A. [1 pt]

10. Find the unique solution $y = y(x)$ to the following initial value problem: [6 pt]

$$\begin{cases} 9y'' + 6y' + y = 4\sin x + 22\cos x\,, \\ y(0) = 2\,, \\ y'(0) = 0\,. \end{cases}$$

Total: 36 pt

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

A

Only for sorting.

Answer form

Calculus 1B – Exam – January 31, 2025

Full name in BLOCK LETTERS _____

Student number _____

Programme _____

⚠ **FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM** ⚠

Question 1.

(a) *Check all that apply:*

(A) ☐ (E) ☐
(B) ☐ (F) ☐
(C) ☐ (G) ☐
(D) ☐ (H) ☐

(b) *Check True or False:*

(A) ☐ True ☐ False
(B) ☐ True ☐ False
(C) ☐ True ☐ False

Question 2. *Fill in your final answer only.*

Question 3. *Fill in your final answer only.*

Question 4. *Fill in your final answer only.*

Radius:

Cartesian coordinates of centre:

Question 5. *Fill in your final answer only.*

(a)

(b)

(c)

Question 6. *Provide a full computation/argumentation.*

Question 7. *Provide a full computation/argumentation.*

Question 8(a). *Provide a full computation/argumentation.*

Question 8(b). *Provide a full computation/argumentation.*

Question 9(a). *Provide a full computation/argumentation.*

Question 9(b). *Provide a full computation/argumentation.*

Question 10. *Provide a full computation/argumentation.*

Additional writing space. *Clearly refer to this space in the original answer*

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