Exam Advanced Logic (211109)

16 April 2013
13:45-17:15

Remarks:

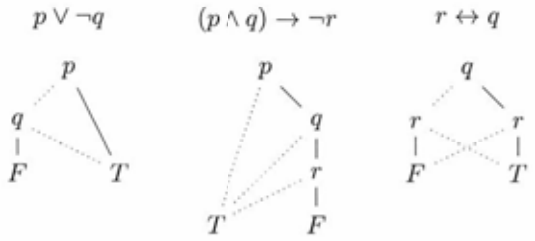
- All exercises contribute equally towards the grade
- Expected time needed: 20 minutes per exercise (average)
- Allowed material: the book and the slides from the lectures. No personal notes!

Exercise 1

Let the set of propositions be given by $\mathcal{P} = \{p, q, r\}$. Show that the following formula is satisfiable, and give all satisfying models:

$$(p \vee \neg q) \wedge (p \wedge q \rightarrow \neg r) \wedge (r \leftrightarrow q)$$

1. Using a semantic tableau;
2. Using a compositional construction of BDDs, where you may start with the following BDDs as building blocks:



Exercise 2

Let the set of all propositions $\mathcal{P}$ consist only of p (i.e., $\mathcal{P} = \{p\}$). Formulas ϕ and ψ are called *logically distinct* if $\phi \not\equiv \psi$. In the following questions, you have to prove your answer; a correct answer consisting only of a number but no proof will not be considered correct.

1. How many pairwise logically distinct propositional formulas do there exist?
2. Consider formulas containing only propositions and the nameless binary operator $\circ_{12}$ in the table on Page 10 (Ed. 2), resp. page 27 (Ed. 3) of the book. How many pairwise logically distinct propositional formulas do there exist in this fragment?

Exercise 3

The following exchange took place some decades ago in the Dutch House of Representatives, between Marcel van Dam, a politician of the PvdA, and a minister belonging to the CDA:

Van Dam Given your personal conviction, do you fit into the government?
Minister Sure: I fit into the CDA, the CDA fits into the government, so I fit into the government.
Van Dam That's nonsense: I fit into my coat, my coat fits into my bag, but I do not fit into my bag!

Intuitively, it is clear that Van Dam refutes the statement of the minister. Formalise the statement of the minister and Van Dam's reply in the framework of predicate logic, and explain how the refutation shows up. (*Hint:* there is an unspoken assumption behind the minister's statement that you have to make explicit.)

Exercise 4

Assume that there exist predicates nat and eq, a function $prod$ and a constant two over the real numbers, with the following interpretation:

- $nat(x)$ expresses the property that x is a natural number;
- $eq(x, y)$ expresses the property that x and y are the same number;
- $prod(x, y)$ is the product of the numbers x and y.
- two stands for the natural number 2.

1. Give a formal definition of the intended interpretation.
2. Pythagoras was reputedly drowned by his students when he proved that the square root of 2 is not a *rational* number; i.e., it is not the result of the division of two natural numbers. This was in fact the end of the Pythagorean school and spelled the doom of number theory for the next two millennia or so.

   Let's call this "Pythagoras' last theorem." State this theorem in terms of the above vocabulary.
3. Give an alternative interpretation under which Pythagoras' last theorem does not hold.

Exercise 5

For a given interpretation, a theory can be satisfied or unsatisfied, and complete or incomplete. Are all four combinations possible? Explain your answer.

Exercise 6

The following is an example of an Aristotelean *syllogism*, an early branch of philosophical logic (far predating predicate logic):

No homework is fun.
Some reading is homework.
Therefore, some reading is not fun.

1. Formulate this as an implication in predicate logic, of the form $(\phi \wedge \psi) \rightarrow \chi$.
2. Prove this implication in the Hilbert proof system, where you may translate the above into $\phi \rightarrow (\psi \rightarrow \chi)$.
3. Show the validity of the implication using a semantic tableau.
4. Show the validity of the implication using resolution.

Exercise 7

1. Write a 2-place Prolog predicate `dup` that duplicates all consecutive elements of a list; i.e., such that `dup([1],X)` results in `X=[1,1]` and `dup([1,2],X)` in `X=[1,1,2,2]`.
2. Demonstrate your program by refuting the goal clause `?- dup([2,3],X).` At each resolution step show the goal clauses and unifying substitutions in the SLD-derivation.