Exam Advanced Logic (211109)

13 April 2010
13:45-17:15

Remarks:

- All exercises weigh equally heavy towards the grade
- Expected time needed: 20 minutes per exercise (average)
- Allowed material: the book and the slides from the lectures. No personal notes!

Exercise 1

Consider the following formula in propositional logic:

$$\neg q \vee \big(\big(((p \vee q) \rightarrow r) \wedge \neg p\big) \rightarrow r\big) \ .$$

Show that this formula is valid:

a. Using a semantic tableau
b. Using resolution

Exercise 2

Systematically construct the BDD of the following formula:

$$(p \wedge q) \rightarrow (p \vee r)$$

from the BDD of the subformulae $p \wedge q$ and $p \vee r$

a. Using the ordering $q < r < p$
b. Using the ordering $p < q < r$
c. Describe and explain the difference in complexity of these constructions.

Exercise 3

Assume that there exist predicates *add*, *prod* and *eq* and constants *one* and *two* over the positive natural numbers, with the following interpretation:

- $sum(x, y, z)$ expresses the property that z is the sum of x and y;
- $prod(x, y, z)$ expresses the property that z is the product of x and y;
- $eq(x, y)$ expresses the property that x and y are equal;
- *one* stands for the natural number 1;
- *two* stands for the natural number 2.

a. Give a formal definition of the intended interpretation

b. The Goldbach conjecture states that every even number larger than 2 is the sum of two primes. Express the Goldbach conjecture in predicate logic, using only the above predicates. (*Hint:* you may use auxiliary predicates, as long as you define them in terms of existing predicates).

Exercise 4

Are the following scenarios possible or not? Explain your answer.

a. The Goldbach conjecture is true, but not provable from the Peano axioms

b. The Goldbach conjecture is false, but provable from the Peano axioms

c. The Goldbach conjecture is neither true nor false

Exercise 5

Using semantic tableaux, show that the following formula is neither valid nor unsatisfiable:

$$\forall x.(\neg\exists y.p(x,y) \rightarrow q(x)) \rightarrow \exists x.q(x) \ .$$

From the tableaux, derive an interpretation that satisfies the formula, and one that falsifies it.

Exercise 6

Prove the validity of the following formula:

$$\forall x.\forall y.(p(x,y) \vee p(y,x)) \rightarrow \forall x.\exists y.p(x,y)$$

a. Using the Gentzen deductive system;

b. Using the Hilbert deductive system (where the $\vee$-subformula is replaced by the equivalent $\neg p(x,y) \rightarrow p(y,x)$, and the $\exists$-subformula by $\neg\forall y.\neg p(x,y)$).

Exercise 7

Prove the validity of the formula in the previous exercise using resolution. Show and explain all your steps.

Exercise 8

Given a Prolog predicate `st(X,Y)` that expresses that `X` is smaller than `Y` (in some total ordering), write a Prolog predicate `merge` that merges two ordered lists into a single ordered list.

Assuming program clauses

```
st(1,2).
st(2,3).
```

demonstrate your program by refuting the goal clause `?- merge([1,3],[2],X).` Show the consecutive goal clauses and unifying substitutions in the SLD-derivation.