Introduction To Mathematics
Exam

Instructions

- Motivate all your answers.

- The use of electronic devices or formula sheets is not allowed.

- Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

- For **Final Answer Questions**, on the answer sheet, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

- For **Open Questions**, on the answer sheet, you provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

- If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

Final answer questions

1. For each of the following statements, determine whether it is true or false.   [2 pt]

   (a) $\{2\} \subseteq \{\varnothing, 2\}$

   (b) $\{2\} \in \{\varnothing, 2\}$

   (c) $\varnothing \subseteq \{\varnothing, 2\}$

   (d) $\varnothing \in \{\varnothing, 2\}$

2. For each of the following statements, state whether it is a tautology, a contradiction, or nei-   [2 pt]
   ther.

   (a) $(p \vee q) \to p$

   (b) $(p \wedge q) \to p$

   (c) $(\neg(p \vee q)) \wedge p$

   (d) $(\neg(p \wedge q)) \wedge p$

Open questions

3. Prove the following statement: [3 pt]

For any integers a, b: if $a + b$ is odd, then ab is even.

4. Use mathematical induction on n to prove that for all $n \in \mathbb{N}$, [4 pt]

$$\sum_{i=1}^{2n} (-1)^i \cdot i = n.$$

5. Consider the set A of 3-digit numbers consisting of digits from $\{1, \ldots, 7\}$, and containing each digit at most once. Provide a full solution, including computations and motivation.

   (a) How many numbers in A contain the digit 1? [1.5 pt]

   (b) How many numbers in A contain the digit 1 or the digit 2 (or both)? [1.5 pt]

Total: 14 pt