Motivate all your answers.

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Final answer

On the answer sheet, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer

You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

1. [2 pt] Find an equation for the plane that contains the points $(-1, 0, -2)$, $(1, 3, -3)$, and $(-3, -1, 1)$.

2. Define the vectors $\mathbf{u} = \langle 4, -2, 4 \rangle$ and $\mathbf{v} = \langle 2, -2, 0 \rangle$.

   (a) [2 pt] Calculate the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

   (b) [2 pt] Calculate the projection of $\mathbf{u}$ onto $\mathbf{v}$.

3. [3 pt] Calculate the following limit:
$$\lim_{x \to 0} \frac{x + \sin(2x)}{x \cos x}.$$

4. Define the function $f \colon \left[-\frac{3}{2}, \infty\right) \to \mathbb{R}$ as follows:

$$f(x) = \sqrt{3x^2 + 2x^3}$$

(a) [2 pt] Calculate $\lim\limits_{x \to 0^+} \dfrac{f(x)}{x}$ and $\lim\limits_{x \to 0^-} \dfrac{f(x)}{x}$, and conclude that $\lim\limits_{x \to 0} \dfrac{f(x)}{x}$ does not exist.

(b) [1 pt] Show that f is not differentiable at 0.

(c) [2 pt] Find all critical points of f on the interval $\left(-\frac{3}{2}, \frac{1}{2}\right)$.

Note: you can use that 0 is the only point in $\left(-\frac{3}{2}, \frac{1}{2}\right)$ at which f is not differentiable.

(d) [2 pt] Find the absolute extreme values of f on the interval $\left[-\frac{3}{2}, \frac{1}{2}\right]$, and specify where the extreme values are attained (that is: specify all x_0 for which $f(x_0)$ is a maximum or minimum value).

Note: you may assume that f is continuous.

5. [3 pt] Calculate

$$\lim_{(x,y) \to (0,0)} \frac{xy^2 + x^2 y}{(x^2 + y^2)\sqrt{x^2 + y^2}},$$

or show that this limit does not exist.

6. [3 pt] Find an equation for the tangent plane to the graph of the function

$$f(x, y) = (x - y)\, e^{x(y-2)}$$

at the point $\left(-1, 2, f(-1, 2)\right)$. Simplify the equation as much as possible.

Total: 22 points