Test Probability Theory, 14 June 2019, 13:45-15:45h
Module Data & Information (201300180)

This test has 6 exercises, a formula sheet and a table of the normal distribution.
You should motivate all your answers.
A simple scientific calculator is allowed, a graphical/programmable calculator is not allowed.

1. Consider a sample space S and two events A and B with probabilities $P(\bar{A} \cap \bar{B}) = 0.2$ and $P(B) = 0.6$. A and B are mutually exclusive.
   (a) Calculate $P(A \cup B)$.
   (b) Calculate $P(A|B)$.
   (c) Are A and B independent events?

2. In a region, five individuals from an animal population thought to be near extinction have been caught, tagged, and released to mix into the population. After they have had an opportunity to mix, a second random sample of 10 of these animals is selected. There are actually 25 animals of this type in the region.
   (a) What is the probability that there is at most 1 tagged animal in the second sample?
   (b) What is the standard deviation of the number of tagged animals in the second sample?

3. The joint probability distribution $P(X = i \text{ and } Y = j)$ of two random variables X and Y is:

$i\backslash j$	-1	0	1
-1	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$
0	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$
1	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

   (a) Determine the probability distribution of X, $E(X)$ and $var(X)$.
   (b) Calculate the correlation coefficient of X and Y.
   (c) Are X and Y independent?
   (d) Calculate $P(X > Y)$.
   (e) Determine the conditional probability function of Y given $X = 0$ and calculate $E(Y|X = 0)$.

4. The random variable X is uniformly distributed on the interval $[1, 10]$.
   (a) Determine the 25th percentile of X.
   (b) Determine the distribution function of X.
   (c) Let $Y = \frac{10}{X}$. Determine the range of Y and the probability density of Y.

5. Intelligence quotients (IQs) of people in a large group are normally distributed with a mean of 100 and standard deviation of 16.
   (a) What is the probability that a random person in the group has an IQ between 115 and 140?
   (b) Construct an interval (a, b), which is symmetric with respect to $\mu = 100$, such that approximately 68% of the people have an IQ in this interval (a, b). (What are a and b?)
   (c) In a group of 64 people, what is the probability that their average IQ is at most 103, $P(\bar{X}_{64} \leq 103)$?

6. The times between two jobs arriving to a computer network are assumed to be independent and exponentially distributed. The mean time between two job arrivals is 10 seconds. X_1 is the time (in seconds) from the start to the arrival of the first job and X_i is the time from arrival of job $i-1$ to job i for $i > 1$.

   (a) Determine $P(X_1 > 12)$ and $P(X_1 > 20 | X_1 > 8)$.

   (b) Give the distribution of $S_{10} = \sum_{i=1}^{10} X_i$ (without proof), $E(S_{10})$ and $var(S_{10})$.

Points:

1			2		3					4			5			6		total
a	b	c	a	b	a	b	c	d	e	a	b	c	a	b	c	a	b	
1	2	2	2	1	3	2	2	2	2	2	2	3	2	1	2	2	3	36

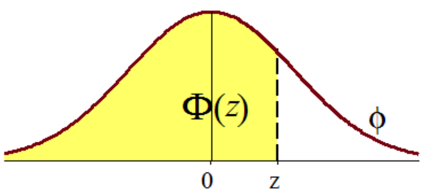
Grade: $\dfrac{\text{number of points}}{36} \times 9 + 1$

Formula sheet Probability Theory for BIT and TCS in module 4

Distribution	$\boldsymbol{E(X)}$	$\boldsymbol{var(X)}$
Geometric	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$n \cdot \frac{R}{N}$	$n \cdot \frac{R}{N} \cdot \frac{N-R}{N} \cdot \frac{N-n}{N-1}$
Poisson $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$, $x = 0, 1, 2, \ldots$	μ	μ
Uniform on $[a, b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Erlang $f_X(x) = \frac{\lambda(\lambda x)^{n-1}e^{-\lambda x}}{(n-1)!}$, $x \geq 0$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$

$$var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} var(X_i) + \sum_{i \neq j}\sum cov(X_i, X_j)$$

Standard normal probabilities



The table gives the distribution function Φ for a N(0,1)-variable Z

$$\Phi(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{x^2}{2}}\, dx$$

Last column: N(0,1)-density function (z in 1 dec.): $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

	Second decimal of **z**										
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**φ(z)**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.3989
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.3970
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.3910
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.3814
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.3683
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.3521
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.3332
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.3123
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.2897
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.2661
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.2420
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	0.2179
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	0.1942
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.1714
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.1497
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0.1295
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0.1109
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0.0940
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0.0790
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0.0656
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0.0540
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0.0440
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0.0355
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0.0283
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0.0224
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0.0175
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0.0136
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0.0104
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0.0079
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.0060
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.0044
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.0033
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.0024
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.0017
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.0012
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.0009
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.0006