Kenmerk: EWI2019/TW/DMMP/MU/Mod7/Exam2

Exam 2, Module 7, Codes 201400483 & 201800141

Discrete Structures & Efficient Algorithms

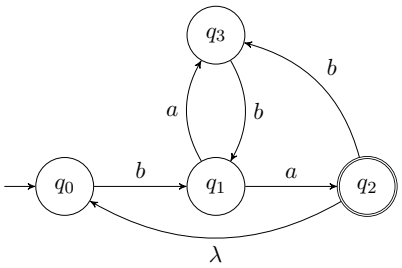
Friday, March 29, 2019, 13:45 - 15:45

All answers need to be motivated. No calculators. You are allowed to use a handwritten cheat sheet (A4, both sides).

This second exam of Module 7 consists of the **L&M part** only, and is a **2h exam**. The total is 50 points. Your exam grade is the maximum of 1 and the total number of points divided by 5, rounded to one digit.

Languages & Machines

1. (11 points) Consider the following NFA with λ-steps M (only q_2 is accepting):



   (a) First eliminate state q_3, adding new transitions labeled with regular expressions, to preserve the accepted language. Show the resulting "expression graph".

   (b) Continue the construction, by eliminating q_1 as well, and read off a regular expression E with $\mathcal{L}(E) = \mathcal{L}(M)$.

   (c) Provide the λ-closure and input-transition function of the automaton M in a table.

   (d) Transform the automaton M in a systematic manner to a (possibly incomplete) DFA.

2. (9 points) Consider the definitions of the following languages over $\Sigma = \{a, b\}$:

   - Language $L_1 := \{a^{2i}\, b^j \mid 0 \leq i \text{ and } 0 \leq j\}$
   - Language $L_2 := \{a^i\, b^j\, a^i \mid 0 \leq i \text{ and } 0 \leq j\}$
   - Language L_3 is an (arbitrary) *finite* language

   Indicate for each of the following languages if they are regular or not. Motivate your answers, either by a proof or a construction.

   (a) Language L_1

   (b) Language L_2

   (c) Language $\overline{L_3} \cup L_3^R$

3. (5 points) Which variables can be derived by chain rules from C in the following grammar G_1? Provide an equivalent grammar without chain rules.

$$G_1 = \begin{cases} S & \rightarrow & A\,B \mid C \\ A & \rightarrow & a\,A \mid B \\ B & \rightarrow & b\,B \mid C \\ C & \rightarrow & c\,C \mid a \mid A \end{cases}$$

4. (5 points) Provide a *regular* grammar, equivalent to the following context-free grammar G_2:

$$G_2 = \begin{cases} S & \rightarrow & B\,S \mid A\,B \\ A & \rightarrow & A\,a\,a \mid \lambda \\ B & \rightarrow & b \end{cases}$$

5. (5 points) Consider the context-free language $L = \{a^i\,b^{i+j}\,c^j \mid i, j > 0\}$.
Give a DPDA (*deterministic* pushdown automaton) for L. Provide a *short* explanation.

6. (5 points) Let G be a context-free grammar in Greibach Normal Form. Let P be a deterministic pushdown automaton, and let E be a regular expression.
Is the language $\mathcal{L}(G) \cup (\mathcal{L}(P) \cap \mathcal{L}(E))$ context-free? (prove)

7. (5 points) Is the class of recursive languages closed under complement? (prove)

8. (5 points) Consider the following Turing Machine with a single tape.
Which language is *decided* by this TM? (explain shortly)