Test Statistics for TCS and BIT, the 1st of February 2019, 8.45-11.00 h.

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This test consists of 4 exercises. The formula sheet and tables are attached. Use of a simple scientific calculator is allowed (no GR). Motivate your answers in a short and clear way.

1. In a study of the economic resilience of metal companies, the order books of the companies are investigated. They are examined for a random sample of 22 metal companies and expressed in the number of weeks it would take to complete the orders in the order book.
The results, in ascending order, are presented in the table below:

6	10	24	27	29	33	35	37	41	47	48
48	51	52	56	57	58	67	70	86	90	94

The classical numerical summary for this sample is determined in the study (the Kurtosis is **not** the SPSS-version "-3"):

Size	Mean	Standard Deviation	Variance	Skewness	Kurtosis
22	48.45	23.49	551.69	0.26	2.83

a. Determine the three quartiles and examine if there are outliers according to the $1.5 \times IQR$-rule.

b. Do you consider a normal model for these measurement data reasonable?
Use the given numerical summary and the result of part a. in your argumentation.

c. A statistical program reported the value of Shapiro Wilk's $W = 0.972$.
Give 1. The coefficient a_2 in the formula of W,
2. The rejection region of Shapiro Wilk's test if $\alpha = 10\%$ and
3. The conclusion you can draw from this result.

d. Determine a 95%-confidence interval for the standard deviation of number of weeks of the order books of metal companies.

e. As you know, $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is an unbiased estimator of σ^2. But is it also the best?
To figure this out, we consider a random sample, drawn from the normal distribution with (unknown) parameters μ and σ^2: it is possible to show that $var(S^2) = \frac{2\sigma^4}{n-1}$. $(n \geq 2)$
This variance of S^2, you may use to solve this exercise.
Express the mean squared error of both $T_1 = S^2$ and $T_2 = \frac{n-1}{n+1}S^2 = \frac{1}{n+1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ in n and σ and determine which of the two estimators of σ^2 is the best.

2. A company institutes an exercise break for its workers to see if it will improve job satisfaction, as measured by a questionnaire that assesses the job satisfaction. Scores of 10 randomly selected workers before and after the implementation of the exercise program are shown in the table below (the higher the score, the more satisfied the worker is):

	Person	1	2	3	4	5	6	7	8	9	10
Job satisfaction scores	Without break	34	28	29	45	26	27	24	15	15	27
	With break	33	36	50	41	37	41	39	21	20	37

a. Can we conclude that exercise breaks improve workers' satisfaction?
Carry out an appropriate **parametric** test (in 8 steps), with α = 1% and state in step 1 of the procedure explicitly what conditions are necessary for this test.

b. Which **non-parametric** test can we use as an alternative of the test in a.? Give for the alternative test only the definition of the test statistic, its observed value and the hypotheses, calculate the p-value of the test and draw the appropriate conclusion in words ($\alpha = 1\%$).

c. If both tests, in a. and in b., can be applied, which test would you recommend? Why?

3. Every year the students of a high school take a physical fitness test during gym classes. One component of the test asks them to do as many push-ups as they can. Below you can find the results of randomly chosen boys (12) and girls (12):

Boys	17	27	31	17	25	32	28	23	25	16	11	34	$\bar{x} = 23.83,\ s_X = 7.21$
Girls	24	7	14	16	2	15	19	25	10	27	31	8	$\bar{y} = 16.50,\ s_Y = 8.94$

a. Give a 95%-confidence interval for the expected difference in number of push-ups of boys and girls. First state all assumptions that are necessary for this interval.

b. Give an interpretation of the interval you found in a.

Give for the questions in parts **c.** and **d.** the appropriate test to answer the question by giving (**only**) the relevant:
1. Test statistic and its observed value,
2. Hypotheses
3. Rejection region for significance level 5% and
4. Your conclusion in words.

c. Can we assume equal variances for the numbers of push ups by girls and boys?

d. Do these observations confirm the gym teacher's statement that, on average, boys can do more push-ups than girls?

4. In July 1991 and again in April 2001, the Gallop Poll asked random samples of 1015 adults their opinion about working parents. The table below summarizes responses to the question "Considering the needs of both parents and children, which of the following do you see as the ideal family in today's society?"

	1991	**2001**
Both work full time	142	131
One works full time, other part time	426	397
One works, other stays home for kids	396	426
No opinion	51	61

Are the opinions on working parents in 1991 and 2001 different?

a. To answer this question, should we conduct a test on independence or a test on homogeneity? Why?

b. Conduct the test you chose in a. at a 5% level of significance. Use the testing procedure.

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Norms: grade $= 1 + \frac{\text{number of points}}{48} \times 9$, rounded to 1 decimal place.

1					2			3				4		Tot
a	b	c	d	e	a	b	c	a	b	c	d	a	b	
4	2	3	3	3	6	4	2	4	1	4	5	1	6	48