UNIVERSITEIT TWENTE.

Tag : Toetsen/ItM+Calc1A.18-19[02].Resit.EN
Course : **Introduction to Mathematics + Calculus 1A**
Date : Friday November 9th, 2018
Time : 13:45 – 16:45

Motivate all your answers.
The use of electronic devices is not allowed.

1. (a) [2 pt] Let $A = \{\frac{k}{k+1} | k \in \mathbb{N}\}$. For each of the following, state whether it exists, and if it does, determine its value:
$\inf A, \max A, \min A, \sup A$.

(b) [3 pt] Use a truth table to show that the statement $(p \to q) \to (p \wedge q)$ is not a tautology, and provide a counterexample.

2. [3 pt] Let $m \in \mathbb{Z}$ and $n \in \mathbb{Z}$. Prove that if the product mn is even, then either m is even, or n is even (or both).

3. [3 pt] Use mathematical induction on n to prove that $\forall n \in \mathbb{N} \cup \{0\}$:

$$\sum_{i=0}^{n} \binom{i+2}{i} = \binom{n+3}{n}$$

Hint: use that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ for all $n \in \mathbb{N}, r \in \mathbb{N}, r \leq n$.

4. Consider the set A of numbers consisting of 4 digits, where each digit is from the set $\{1, 2, 3\}$. For example $1311 \in A$.

(a) [1 pt] How many odd numbers are there in A?

(b) [2 pt] How many numbers in A are either odd, or start with the digit 1 (or both)?

5. Define $P(1, -1, 0)$, $Q(2, 0, 4)$, and $R(2, 3, 1)$.
Let $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$.

(a) [1 pt] Calculate $\mathbf{u} \times \mathbf{v}$.

(b) [2 pt] Find an equation of the plane that contains the points P, Q and R.

(c) [2 pt] Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

(d) [2 pt] Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

6. [2 pt] Calculate

$$\lim_{x \to 0} \frac{x\, e^x}{e^{2x} - 1}.$$

7. Define

$$f(x) = \begin{cases} x \sin\left(\ln|x|\right) & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$$

0,5 (a) [2 pt] Show that $\lim_{x \to 0} f(x) = 0$.

(b) [1 pt] Is f continuous at 0? Motivate your answer.

8. The function $f\colon (-\infty, 3] \to \mathbb{R}$ is defined by

$$f(x) = \sqrt{3x^2 - x^3}.$$

(a) [2 pt] Using the definition of the derivative, show that f is not differentiable at 0.

1/4 (b) [2 pt] Find all critical points of f.

0,5 (c) [1 pt] Find the absolute extreme values of f on the interval $[1, 3]$.

0,5 9. [2 pt] Calculate

$$\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{\sqrt{x^2 + y^2}},$$

or show that this limit does not exist.

1 10. [3 pt] Find an equation for the tangent plane to the graph of the function

$$f(x, y) = \ln\left(x^2 + y^2 - 1\right)$$

at the point $\left(1, -1, f(1, -1)\right)$.

Simplify the equation as much as possible.

Total: 36 points