

Joven Kock.

Course : **Introduction to Mathematics**

Date : September 21, 2018

Time : 13.45 – 14.45 hrs

**Motivate all your answers.
The use of electronic devices is not allowed.**

1. For $U = [1, 6]$, let $A = \{x \mid x \text{ is even}\}$ and $B = \{6\}$.
 - (a) [1 pt] Determine $A \cap B$.
 - (b) [1 pt] Determine $A \cup B$.
 - (c) [1 pt] Determine \overline{B} .

2. [4 pt]
Consider the statements $S_1 : ((p \wedge q) \rightarrow p) \rightarrow q$ and $S_2 : (p \wedge q) \vee (\neg p \wedge q)$.
Use a truth table to determine whether S_1 and S_2 are logically equivalent.

3. [4 pt]
We place $2n + 1$ items in n boxes. Prove by mathematical induction over n that for all $n \in \mathbb{N}$ there is at least 1 box with at least 3 items.

4. In this exercise, you only need to give the correct expression; you don't need to calculate the exact value. For example, you don't need to simplify $(20!)/(3!)$.
 - (a) [1 pt] Consider numbers that consist of exactly 100 digits, and for which each digit is an element of the set $\{1, 2, 3, 4\}$. How many such numbers exist?
 - (b) [2 pt] Consider numbers that consist of exactly 100 digits, for which each digit is an element of the set $\{1, 2, 3, 4\}$, and for which exactly 50 digits are even. How many such numbers exist?

Total: 14 points

