Introduction to Mathematical Analysis
(201600166 - 2EC, 201600167 - 4EC)

Friday 27 October 2017,
13:45-14:45 (2EC) or 13.45-15.45 (4EC)

201600166 - 2EC: exercises 1, 2 and 3 (total $18+2$ points), 1 hour.
201600167 - 4EC: exercises 1, 2, 3, 4, 5 and 6 (total $36+4$ points), 2 hours.

State clearly for which course you take the exam!

Motivate all your answers and computations.
The use of electronic equipment is not allowed.

Exercise 1 (6p)

Suppose P, Q, R and S are statements.

a. **(2p)** Suppose P is false and that the statement $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true. Find the truth values of R and S. (This can be done without a truth table.)

b. **(4p)** Decide whether or not the following pairs of statements are logically equivalent:

$$(\neg P) \wedge (P \Rightarrow Q)$$

and

$$\neg(Q \Rightarrow P).$$

Exercise 2 (6p)

Given

$$\begin{cases} a_{n+1} & = \sqrt{6 + a_n} \\ a_1 & = 1. \end{cases}$$

Prove by induction that $a_{n+1} \geq a_n$ holds for all $n \in \mathbb{N}$.

Exercise 3 (6p)

Prove or give a counterexample for the following statements:

a. **(2p)** Suppose a, b and c are integers. If $a^2|b$ and $b^3|c$, then $a^6|c$.

b. **(2p)** Let $n \in \mathbb{Z}$. If $6 \nmid n^2$, then $6 \nmid n$.

c. **(2p)** There exist no integers a and b for which $21a - 30b = 1$.

Turn for more.

Exercise 4 (6p)

a. **(2p)** State the Intermediate Value Theorem (IVT) for a function $f : [a, b] \to \mathbb{R}$.

b. **(4p)** Given a continuous $f : \mathbb{R} \to \mathbb{R}$ for which $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$.

Prove that there is a $c \in \mathbb{R}$ such that $f(c) = \sin(c)$.

Exercise 5 (6p)

Given is the function $F(x) = \int_1^x \frac{\sqrt{t^3+1}}{t}\,\mathrm{d}t$.

a. **(3p)** Find the Taylor polynomial of order 2 generated by F centered at $x = 1$.

b. **(3p)** Show that $\lim_{x\to\infty} F(x)$ does not exist.

Exercise 6 (6p)

a. **(3p)** Determine whether the series is converging or diverging:

$$\sum_{k=1}^{\infty} \frac{k^3}{2^k}$$

b. **(3p)** Find the interval of convergence of the power series:

$$\sum_{k=1}^{\infty} \frac{x^k}{k \cdot 3^k}$$

The End.