Kenmerk: EWI2020/TW/MOR/MU/Mod7/Re-Exam1

Re-Exam 1, Module 7, Codes 201400483 & 201800141

Discrete Structures & Efficient Algorithms

Tuesday, July 14, 2020, 08:45 - 11:45

All answers need to be motivated, arguments and proofs must be complete, and reference to online sources is not sufficient. You are allowed to use the textbook(s), lecture slides, as well as your handwritten cheat sheet per topic (ADS, DM) during the exam.

This exam consists of two parts, with the following (estimated) times per part:

Algorithms & Data Structures (ADS)	ca. 1h	(30 points)
Discrete Mathematics (DM)	ca. 2h	(60 points)

The total is 30+60=90 points. Your grade is $1 + 0.1x$, x being the number of points, rounded to one digit. That means, you need 45 points to get a 5.5.

Please read carefully: By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test. In case of doubt, it might be that we have to decide not to count the test result, which could include invalidating the test results of all other students, too. In case of doubt, we take the liberty of taking oral examinations afterwards. Therefore, our appeal is to your own responsibility:

You maximise your own, and all your fellow students' chance to have this test result remain valid, by adhering to the rules as stated below.

In order for the test to be graded, the following text must be copied on the first page of your solutions:

> **"I have made this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test" [Name, Student no., Location, Date, Signature].**

Please upload one pdf file for this ADS part of (Re)Exam 1.

Algorithms & Data Structures

1. (10 points)

   (a) Consider this sorting algorithm that sorts from a sequence A of integers the segment $A[l, \ldots, r]$ where $1 \leq l \leq r$:

```
def sort(A,l,r):
    if l<r-1 :
        k=(r-l+1)//3
        sort(A,l,r-k)
```

```
        sort(A,l+k,r)
        sort(A,l,r-k)
    else :
        if A[l]>A[r] : A[l],A[r]=A[r],A[l]
```

Give a recurrence relation for the time complexity of this algorithm, expressed in the number of comparisons.

(b) Give the asymptotic order of the solution of the following recurrence equation:

$$T(n) = 3 \cdot T(\frac{n}{3}) + 3,\ T(1) = 1$$

2. (a) (5 points)
   Someone claims to have implemented a priority queue in such a way that *insert(e,k)*, *getMin()*, and *delMin()* all have complexity $\Theta(1)$. Show that this cannnot be true (hint: think about sorting).

   (b) (5 points)
   Given a completely filled binary tree of depth 3, where each node has as a (unique) key one of the letters A, B, ..., or O, in such a way that the tree is alphabetically sorted pre–order.
   What is the order in which you encounter the letters if you traverse this tree in an in–order way?

3. (10 points)

   A character string can be transformed into another string using just insertions and deletions. For instance, COST can be transformed into CAT by deleting the O and S and inserting the A. The minimum number of deletions and insertions for transforming string u into string v is called the LCS distance between u and v (so the LCS distance between COST and CAT is 3).

   We assume string u is given by an array indexed from 1 to n, and v by an array indexed from 1 to m. Define $D[i,j]$ as the LCS distance between $u[1..i]$ and $v[1..j]$ (i.e. the prefixes of u and v of length i respectively j).

   (a) Explain that $D[i,j]$ satisfies the recursive equation
   $D[i,j] =$
   - i if $j = 0$
   - j if $i = 0$
   - $D[i-1,j-1]$ if $u[i] = v[j]$
   - $1 + min\{D[i-1,j], D[i,j-1]\}$ otherwise

   (b) Give an algorithm that computes the LCS distance between u and v. The algorithm should have complexity $\Theta(mn)$ where n is the length of u and m the length of v.

Discrete Mathematics

Please see other pdf for DM questions.